Student Workshop Integrability 2023

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Book of Abstracts

Contents

2

Partial integrability: from three body to many body

Author: Zhao Zhang[1]

[1] *SISSA*

Novel paradigms of ergodicity breaking have been mushrooming in recent years, most notably in the form of quantum many-body scars. Here I present yet another mechanism of weak ergodicity breaking, in a partially integrable spin chain. Breakdown of integrability in the generic subspaces is manifested with the violation of the Yang-Baxter equation for scattering matrices, but we were nevertheless able to construct antisymmetric and symmetric basis that reveal integrable excited states in the framework of coordinate Bethe Ansatz. These Bethe states in disguise distinguish themselves dynamically from the previously known partial integrability due to Hilbert space fragmentation, as initial states can overlap with both integrable eigenstates and ETH satisfying chaotic ones which leads to slow thermalization.

5

Entanglement Hamiltonian in the Domain Wall melting

Author: Federico Rottoli[1]

Co-authors: Stefano Scopa [1]; Pasquale Calabrese [1]

[1] *SISSA*

The domain wall melting offer a paradigmatic example of an out-of-equilibrium problem. In the past years the application of many techniques has made possible to investigate the entanglement properties of this problem, by allowing the computation of the entanglement entropies. In this work we move forward, by deriving the Entanglement Hamiltonian (the logarithm of the reduced density matrix) in the melting of a domain wall in the free Fermi chain. With respect to other measures of entanglement, the Entanglement Hamiltonian offer a more complete, operatorial characterisation. We conduct an exact numerical study on the lattice, showing how to recover the field theoretical prediction with the use of a careful limiting procedure.

6

Inelastic decay from integrability

Authors: Amir Burshtein[None]; Moshe Goldstein[1]

[1] *Tel Aviv University*

A hallmark of integrable systems is the purely elastic scattering of their excitations. Such systems posses an extensive number of local conserved charges, leading to the conservation of the number of scattered excitations, as well as their set of individual momenta. In this talk, I will show that inelastic decay can nevertheless be observed in circuit QED realizations of integrable boundary models. I will consider scattering of microwave photons off impurities in superconducting circuits implementing the boundary sine-Gordon and Kondo models, which are both integrable. I will show that not only inelastic decay is possible for the microwave photons, in spite of integrability, and thanks to a nonlinear relation between them and the ellastically-scattered excitations, but also that integrability in fact provides strong analytical tools allowing to obtain exact expressions for response functions describing the inelastic decay, using the framework of form factors. I will show how one may obtain not only the total inelastic decay rate of the microwave photons, extracted from a 2-point response

function, but also go beyond linear response and obtain the energy-resolved inelastic decay spectrum, using a novel method to evaluate form factor expansions of 3-point response functions which could prove useful in other applications of integrable quantum field theories. Our results compare quantitatively well with measurements from the Manucharyan group, and are instrumental in shedding light on experimental data that provide evidence for the elusive Schmid-Bulgadaev dissipative quantum phase transition.

7

A large family of IRF solvable lattice models based on WZW models

Authors: Juan Ramos Cabezas[1]; Vladimir Belavin[1]; Doron Gepner[2]

[1] *Ariel University*

[2] *Weizmann Institute*

In this talk, we will discuss a large class of Interaction-Round-a-Face (IRF) solvable lattice models that are based on the symmetry algebras of WZW models, namely, the affine Lie algebras $\hat{\mathfrak{su}}(n)$, $\hat{\mathfrak{so}}(2n+1)$, $\hat{\mathfrak{sp}}(n)$, $\hat{\mathfrak{so}}(2n)$, and $\hat{G}_2$ (studied by Jimbo et al., and Kuniba, respectively). We have derived a general formula for the so-called crossing multipliers of these models and have shown (in a published paper), that these crossing multipliers can be expressed using the principally specialized characters of the algebras in question. Based on this result, we will argue that the crossing multipliers in a large class of solvable IRF lattice models can be obtained from the characters of the conformal field theory on which they are based and will discuss the relevance of this idea in the computation of the Local State Probabilities. We will also explain the generalization (this is part of a work in progress) of these models to certain quotients of the original groups. In particular, we will talk about the $SU(3)/\mathbb{Z}_3$ quotient.

8

Finite temperature spin diffusion in the Hubbard model in the strong coupling limit

Author: Arthur Hutsalyuk[1]

Co-authors: Oleksandr Gamayun [2]; Pozsgay Balázs [3]; Mikhail Zvonarev [4]

[1] *Eötvös Loránd University (ELTE)*

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[4] *Université Paris-Saclay*

We investigate finite temperature spin transport in one spatial dimension by considering the spin-spin correlation function of the Hubbard model in the limiting case of infinitely strong repulsion. We find that in the absence of bias the transport is diffusive, and derive the spin diffusion constant. Our approach is based on asymptotic analysis of a Fredholm determinant representation, which is entirely analytic and free of phenomenological assumptions of Generalized Hydrodynamics.

9

Bethe bound states and spin transport

Author: Andrew Urilyon (Urichuk)[1]

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Spin transport in the paramagnetic XXZ model exhibits simultaneous ballistic and diffusive transport at all non-zero temperatures, which stems from the partial conservation of the local spin current. At T=0 the absence of Bethe bounds states, or strings, yields purely ballistic spin transport, however investigations at non-zero temperatures suggest that the Bethe strings play a key role. High temperature properties of the non-linear spin transport further supports this idea that the bound states must be understood to account for non-zero temperature transport. This raises many questions about what form a low temperature field theoretical description should take, since standard non-linear Luttinger liquid methods predict a ballistic transport temperature scaling inconsistent with the BA results. This presentation aims to collate different methods and by comparing them suggest future directions for investigation of both ballistic and diffusive transport at all temperature regimes in the XXZ model.

10

Wilson lines construction of sl3 toroidal conformal blocks

Authors: Pietro Oreglia[1]; Vladimir Belavin[1]; Juan Ramos Cabezas[1]

[1] *Ariel University*

We study $\mathcal{W}_3$ toroidal conformal blocks for degenerate primary fields in AdS/CFT context.
In the large central charge limit $\mathcal{W}_3$ algebra reduces to $\mathfrak{sl}_3$ algebra and $\mathfrak{sl}_3$ blocks are defined as contributions to $\mathcal{W}_3$ blocks coming from the generators of $\mathfrak{sl}_3$ subalgebra.
We consider the construction of $\mathfrak{sl}_3$
toroidal blocks in terms of Wilson lines operators of $3d$ Chern-Simons gravity in the thermal AdS_3 space-time. According to the correspondence,
degenerate primary fields are associated with Wilson lines operators acting in the corresponding finite-dimensional $\mathfrak{sl}_3$ representations. We verify
this dual construction
for one-point toroidal block using
$\mathfrak{sl}_3$ tensor technique in the bulk theory and an algorithm based on AGT correspondence in the boundary CFT.

11

Entanglement evolution after a global quench across a conformal defect

Authors: Luca Capizzi[None]; Viktor Eisler[None]

We consider the dynamics of a one-dimensional quantum system in the presence of a localized defect. We prepare the system in a short-range entangled state, we let it evolve ballistically, and we study the entanglement across the defect. Linear growth of the entanglement entropy is observed, whose slope depends both on the scattering properties of the defect and the initial state. The protocol above is characterized by Conformal Field Theory, and the Rényi entropies are related to the correlation functions of twist fields in a bounded two-dimensional geometry. Moreover, we investigate a particular lattice realization in a free-fermion chain, giving a prediction for the linear slope via a quasi-particle picture.

12

The analytic structure of non-perturbative corrections in integrable field theories

Author: Istvan Vona[1]

[1] *Wigner RCP, ELTE University*

Several one-dimensional models (both relativistic and non-relativistic) can be solved - at least numerically - by a linear, thermodynamic Bethe ansatz-like integral equation in specific settings. Excellent examples are the nonlinear sigma models in an external field coupled to a conserved charge or the Lieb-Liniger/Gaudin-Yang models for different couplings. In these scenarios, it is possible to perturbatively expand the physical quantities (e.g., the energy density of the particles) in some specific parameter up to very high orders by using the integral equation. This expansion is an asymptotic series that contains lots of hidden analytic information on the exact result. In some cases, this information is complete, and it is possible to reconstruct the exact result from the perturbative series only using resummation techniques. That typically means summing up an infinite number of exponentially suppressed non-perturbative corrections accessible from the perturbative data by resurgence theory. In other cases, there is a mismatch between the exact result and the abovementioned procedure, which can be explained by a careful analysis of the integral equation in Fourier space. One can find the missing family of exponential corrections as well, acquiring a complete understanding of the analytical structure of the physical quantity in terms of the expansion variable, at least from the mathematical point of view. The aim of the talk is a brief summary of this method based on arXiv:2212.09416.

13

Temporal Entanglement in Chaotic Quantum Circuits

Author: Alessandro Foligno[1]

Co-authors: Tianci Zhou [2]; Bruno Bertini [1]

[1] *University of Nottingham*

[2] *Massachusetts Institute of Technology*

The concept of space-evolution (or space-time duality) has emerged as a promising approach for studying quantum dynamics. The basic idea involves exchanging the roles of space and time and evolving the system using a space transfer matrix whose fixed points, also known as influence matrices, describe the interaction of the rest of the system acting as a bath on a subsystem. To evaluate the potential of this method as a numerical scheme, it is important to understand whether the influence matrices can be efficiently encoded in a classical computer. It is then natural to wonder what is the scaling of their entanglement, dubbed temporal entanglement, as a function of time. In this talk, I am going to present these ideas, and show how they can be applied for the calculation of generic two points correlation functions. Moreover, the Renyi entropies of these fixed points have a different qualitative behavior than the Von Neumann, which can be characterized exactly in some marginal cases, similarly to what has been observed for regular entanglement in systems with conservation laws.
This talk is based on the following paper: arXiv:2302.08502

14

A Generalized hydrodynamics approach to impurities in integrable models: Mesoscopic impurities

Authors: Friedrich Huebner[1]; Benjamin Doyon[1]

[1] *King's College London*

It is well established that the large-scale behaviour of integrable models is captured by Generalized Hydrodynamics (GHD). Inserting an impurity into an integrable model typically breaks integrability and therefore its effect is hard to analyse analytically. In this talk I will first briefly introduce a GHD viewpoint on impurities: An impurity is given by a boundary condition in the GHD equation, which relates the ingoing and outgoing currents at the position of the impurity.
Unfortunately, deriving this GHD boundary condition for a specific microscopic impurity requires a full solution of the microscopic impurity and is therefore analytically not possible in general. In the second part of my talk I will focus on impurities whose spatial size L_{impurity} is mesoscopic $L_{\mathrm{micro}} \ll L_{\mathrm{impurity}} \ll L_{\mathrm{system}}$, which is an approximation for large impurities. The large size of the impurity allows to still assume the usual requirements for GHD, like local equilibrium, also at the impurity. This means that the impurity can be fully described using the GHD framework. I will show that the impurity problem reduces to solving the stationary GHD equation and discuss the physical consequences of the approximation. Furthermore, I will explain a general strategy to approach the stationary GHD equation that, in simple cases, allows to find full analytical solutions to the scattering problem.

15

Yang-Baxter deformations of the flat space string

Author: Khalil Idiab[1]

[1] *HU Berlin*

The study of symmetric space sigma models and their integrable Yang-Baxter deformations in the context of the AdS/CFT correspondence has led to many important results. However, the quantization of these models remains a challenging problem. To gain new insights into the quantum structure of these models, we propose to study Yang-Baxter deformations of the flat space string. The advantage of flat space is that it allows us to perform certain calculations explicitly, which can serve as a check to some of the known theory.
In this talk, we will present our approach to extending Yang-Baxter deformations to flat space and we will demonstrate an explicit check of some known results.

16

The classical sine-Gordon model and its hydrodynamics

Author: Alvise Bastianello[1]

Co-author: Rebekka Koch

[1] *Technical University of Munich*

The sine-Gordon field theory is a paradigmatic integrable model that shows up in the most diverse contexts, emerging as the low-energy description of a wealth of systems. Depending on the experimental platform, the sine-Gordon realization may be close to its classical limit: this is the case, for example, in the interference pattern of two weakly-coupled quasicondensates, as it is realized in

Vienna. This observation motivates us in studying the classical model.
In this talk, I will present the thermodynamics and generalized hydrodynamics of the classical sine-Gordon field theory. In the stream of the talk, I will touch several points: I will discuss why previous works based on soliton-gas picture failed to correctly capture the sine-Gordon's thermodynamics, I will show how to obtain the correct result by taking the semiclassical limit of the quantum model, and build its generalized hydrodynamics. If time permits, I will use this toolbox to discuss experimentally-motivated setups.

17

Matrix Product Symmetries and Breakdown of Thermalization

Authors: Márton Borsi[1]; Balázs Pozsgai[None]; Levente Pristyák[None]

[1] *Eötvös Loránd University, Budapest*

The presence of unconventional symmetries in quantum systems can result in ergodicity breaking and the prevention of the usual thermalization process. One example is the class of Hilbert space fragmented models which possess an exponentially growing number of kinematically disconnected sectors in the Hilbert space. The phenomenon originates from a symmetry algebra of similarly increasing dimension and can be present in both integrable and non-integrable systems.
In my talk I introduce a mechanism for Hilbert space fragmentation: I present spin chains constructed by hard rod deformation that possess a non-commutative symmetry algebra given by Matrix Product Operators (MPO). As a result, persistent oscillations appear in non-equilibrium situations. Extra symmetries in integrable models are commonly described by MPOs, but in our work we also find non-integrable examples, thus providing a new way of ergodicity breaking.

18

Current mean values in the XYZ model

Author: Levente Pristyák[None]

The theory of Generalized Hydrodynamics (GHD) was introduced in order to describe the non-equilibrium transport properties of integrable quantum systems. It builds on the (infinitely many) continuity equations, that can be written for the conserved quantities present in these systems. Even though GHD itself does not rely on particle conservation on a fundamental level, all previous works treated U(1) symmetric models. Therefore, it would be interesting to work out the theory for models lacking U(1) symmetry. As a first step in this direction, in my talk I consider the mean values of the current operators in the spin-1/2 XYZ chain. Using a generalized Algebraic Bethe Ansatz and the algebraic construction of the current operators, I derive an exact formula for these current mean values in finite size, with a functionally identical form to that of the earlier results for the XXZ chains. The talk is based on the paper arXiv:2211.00698.

19

Symmetry resolution of entanglement measures in excited states of (1+1)dimensional massive integrable quantum field theories

Author: Michele Mazzoni[1]

[1] *City University of London*

In this talk I will present the results obtained with my supervisor Dr. Olalla Castro-Alvaredo and other collaborators on the symmetry resolution of entanglement measures in excited states of (1+1)d massive integrable quantum field theory (IQFT). This work generalises the results known to hold for excited states of massive IQFT with no internal symmetry to the case in which the theory enjoys a U(1) symmetry and the entanglement entropy (EE) admits a charge decomposition. Specifically, we looked at a complex free boson and a complex free fermion on a circle. The results are obtained through a field-theoretic approach that makes use of a "replica trick" and generalised twist fields. Becuse of the universality features of our formulae, we found that these can be derived also in a much simpler setup, in which the excitations are multi-qubit states. Our theoretical predictions perfectly matches the numerical analysis performed on two different lattice Hamiltonians. Finally, I will briefly present some further generalisations of this work to higher dimensional theories, interacting/non integrable theories and to other entanglement measures.

20

GUE-corners process in the Aztec diamond

Author: Nicolas Robert[1]

[1] *UCLouvain*

Random tiling problems, or perfect matchings, constitute a certain class of exactly solvable models studied by both mathematicians and mathematical-physicists since the 20-th century. These models can be viewed as a playground where some universal behaviours take place, and they are particularly interesting as much for their links with other statistical models as for their rich mathematical structure.
In this presentation, I will speak about the Aztec diamond model which exhibits an arctic phenomenon (i.e. spatially separated regions with different behaviours) similar to what happens in the 6-vertex model with domain-wall boundary conditions. More specifically, I will focus on a specific determinantal point process occurring in this model and show its quite surprising link with the GUE-corners distribution coming from the Random Matrix Theory.

21

The spin-1 XYZ chain and related vertex models

Author: Sandrine Brasseur[1]

[1] *UCLouvain*

In this talk, we are interested in an integrable spin-1 XYZ chain with twisted boundary conditions. We show that the XYZ Hamiltonian possesses the remarkably simple eigenvalue E=0. In some regime, it is conjectured to be the ground-state eigenvalue. Moreover, we express a sum rule involving the zero-energy states in terms of special polynomials. These polynomials have connections to other problems in integrable systems and enumerative combinatorics. To obtain our results, we follow an idea of Baxter's, and consider a related eighteen-vertex SOS model. There, computations are performed using Sklyanin's quantum Separation of Variables. This talk is based on joint work with Christian Hagendorf.

23

Factorization of density matrices in the critical RSOS models

Authors: Holger Frahm[1]; Maxime Großpietsch[None]; Hannes Kakuschke[None]; Daniel Westerfeld[None]

[1] *Leibniz Universität Hannover*

We study reduced density matrices of the integrable critical RSOS model in a particular topological sector containing the ground state. Similar as in the spin-$1/2$ Heisenberg model correlation functions of this model on short segments can be 'factorized': they are completely determined by a single nearest-neighbour two-point function ω capturing the dependence on the system size and the state of the system and a set of structure functions. The latter can be expressed in terms of the possible operators on the segment, in the present case representations of the Temperley-Lieb algebra TL_n, and are independent of the model parameters. We present explicit results for the function ω in the infinite system ground state of the model and compute multi-point local height probabilities for up to four adjacent sites for the RSOS model and the related three-point correlation functions of non-Abelian $su(2)_k$ anyons.

Poster Session I / 24

Prethermalization in coupled one-dimensional gases

Authors: Maciej Łebek[1]; Miłosz Panfil[1]; Robert Konik[2]

[1] *Faculty of Physics, University of Warsaw*

[2] *Brookhaven National Laboratory*

We consider the problem of the development of steady states in one-dimensional Bose gas tubes modelled by the integrable Lieb-Liniger model. The tubes are weakly coupled to one another through a density-density interaction, which weakly breaks the integrability of the system. We analyze this development through a Boltzmann collision integral approach. We argue that when the leading order of the collision integral, where single particle-hole excitations are created in individual gases, is dominant, the state of the gas evolves first to a non-thermal fixed point, i.e. a prethermalization plateau. This order is dominant when a pair of tubes are inequivalent with, say, different temperatures or different effective interaction parameters, γ. We characterize this non-thermal prethermalization plateau, constructing both the quasi-conserved charges that control the existence of this plateau as well as the associated generalized Gibbs ensemble.

25

Dynamical universality of charged single-file systems and integrable spin chains

Author: Žiga Krajnik[1]

[1] *University of Ljubljana*

We introduce and discuss dynamical universality of charge fluctuations in charged single-file systems. The full counting statistics of such systems out of equilibrium generically undergoes first and second order dynamical phase transitions, while equilibrium typical fluctuations are given by a universal non-Gaussian distribution. Similar phenomenology of dynamical criticality is observed in equilibrium in the easy axis and isotropic regimes of an integrable spin chain. While the easy axis regime does not satisfy a single-file kinetic constraint, it nevertheless supports the non-Gaussian distribution of the charged single-file universality class. Fluctuations at the isotropic point are also anomalous and distinct from those of the Kardar-Parisi-Zhang universality class.

26

The arctic curve of the four-vertex model

Author: Andrea Maroncelli[1]

[1] *University of Florence*

We consider the four-vertex model, which is a special case of the six-vertex model in which two vertices are set to zero. Under specific choices of fixed boundary conditions, this model exhibits spatial phase separation, between frozen and disordered regions, sharply separated by a smooth curve, known as arctic curve. The most interesting aspect of the four-vertex model is that, even though it is interacting, it is still exactly solvable in a relatively simple way. Here we use the Tangent Method, which is an exact, although heuristic method, to compute the arctic curve.

27

Relaxation dynamics in Integrable Field Theories

Author: Emanuele Di Salvo[1]

[1] *Utrecht University*

Out of equilibrium dynamics of integrable systems have been intensively studied in the past 20 years. However, a full characterisation of time evolution of an integrable field theory after a quantum quench is still missing. We investigate many processes occurring during relaxation towards a steady state and describe them in terms of analytical properties of form factors of operators in the post-quench theory. As an example, results for thermal Ising, Sinh-Gordon and Yang-Lee field theories are shown. We extend this approach to non-relativistic theories by mapping them via out-of-equilibrium non-relativistic limit.

28

Sigma-models as Gross-Neveu models

Author: Viacheslav Krivorol[1]

Co-author: Dmitri Bykov [2]

[1] *Institute of Theoretical and Mathematical Physics and Steklov Mathematical Institute, Moscow*

[2] *Steklov Mathematical Institute of Russian Academy of Sciences and Institute of Theoretical and Mathematical Physics*

We consider the reformulation the sigma-models as a generalized Gross-Neveu models. In fact, this representation crucially simplifies the analisys of quantum aspects of sigma-models. We show, as simplest application, that in this formulation the derivation of the β-function of the sigma-models reduces to ordinary calculation of Feynman diagrams (as in ϕ^4 model), while in the usual formulation one need more complicated tools (background field method, for example). We construct the Gross-Neveu representation for the sigma models, which target spaces are $\mathbb{CP}^n$ and Grassmannians (include orthogonal and symplectic one). We also consider the connection of this representation with algebraic and complex geometry, theory of nilpotent orbits and integrability (potentially, on Riemann surfaces).

31

Welcome Adress

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Formal begin of workshop. Introduction to the venue, schedule and giving other important information.

32

Hydrodynamics and fluctuations in integrable and quasi integrable systems

33

Long range interacting models and integrability

35

Analytical results for the entanglement dynamics of disjoint blocks in the XY chain

Author: Riccarda Bonsignori[1]

[1] *Ruder Boskovic Institute (Zagreb)*

The study of the dynamics of entanglement measures after a quench has become a very active area of research in the last two decades, motivated by the development of experimental techniques. However, exact results in this context are available in only very few cases. In this talk, I present the results of a work done in collaboration with Gilles Parez, in which we provide the proof of the quasiparticle picture for the dynamics of entanglement entropies for two disjoint blocks in the XY chain after a quantum quench. As a byproduct, we also prove the quasiparticle conjecture for the mutual information in that model. Our calculations generalize those presented in [M. Fagotti, P. Calabrese, Phys. Rev. A 78, 010306 (2008)] to the case where the correlation matrix is a block-Toeplitz matrix, and rely on the multidimensional stationary phase approximation in the scaling limit. We also test the quasiparticle predictions against exact numerical calculations, and find excellent agreement. In the case of three blocks, we show that the tripartite information vanishes when at least two blocks are adjacent.

36

Adiabatic eigenstate deformations and weak integrability breaking of Heisenberg chain

Author: Anastasiia TIUTIAKINA[1]

[1] *Laboratoire de Physique Th´eorique et Mod´elisation*

We consider the spin-1/2 XXX chain weakly perturbed away from integrability by an isotropic next-to-nearest neighbor exchange interaction. Recently, it was conjectured that this model possesses an infinite tower of quasiconserved integrals of motion (charges) [D. Kurlov et al., Phys. Rev. B

105, 104302 (2022)]. In this work we first test this conjecture by investigating how the norm of the adiabatic gauge potential (AGP) scales with the system size, which is known to be a remarkably accurate measure of chaos. We find that for the perturbed XXX chain the behavior of the AGP norm corresponds to neither an integrable nor a chaotic regime, which supports the conjectured quasi-integrability of the model. We then prove the conjecture and explicitly construct the infinite set of quasiconserved charges. Our proof relies on the fact that the XXX chain perturbed by next-to-nearest exchange interaction can be viewed as a truncation of an integrable long-range deformation of the Heisenberg spin chain.

37

Integrable supersymmetric quantum circuit

Author: Pietro Richelli[1]

[1] *University of Amsterdam*

The Floquet Baxterization was introduced as a bridge between quantum integrable models and quantum circuits. In this work we revisit the Floquet Baxterization extending it to graded **R** matrices, introducing an integrable supersymmetric brick-wall quantum circuit. The brick-wall circuit is made up by the **S**-matrix of a supersymmetric particle theory in 1+1 dimensions as unitary gates. The analysis will focus on the limit between the inhomogeneous and homogeneous brick-wall, uncovering some of the rich structure hidden in the quantum circuit. We consider two limits for the **S**-matrix, the first with equally massive particles that will give rise to the Kitaev chain at the critical point. The second with particle with different masses that will give rise to an extended dimerized Kitaev Chain.