Digital Quantum Simulation and Symmetry Protection with Trapped Ions

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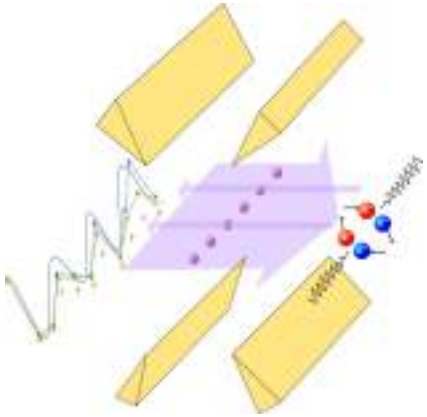
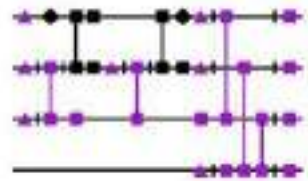
September 25, 2023



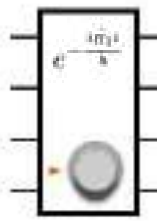
Quantum Simulation

"digital" quantum simulation



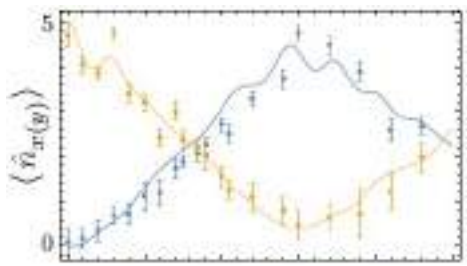
quantum computer
(universal)

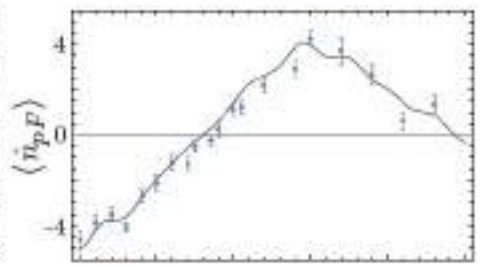
"analog" quantum simulation



tailored native evolution

approximate





simulat**ing** system
(physical system at hand)

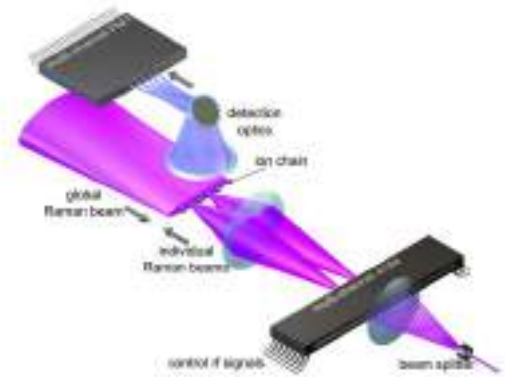
simulat**ed** system
(target dynamics of interest)

Overview

Experimental system

Individually-addressed $^{171}Yb^{+}$ ions

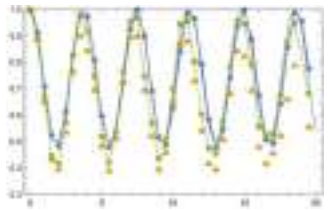
Modular operations and compiler (5-9 qubits)



Applications

Digital Quantum Simulation

The Schwinger model



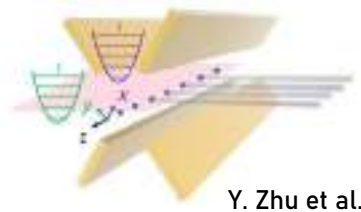
N. H. Nguyen et al., PRX Quantum 3, 020324 (2022).

Hybrid Quantum Simulation

The Yukawa model

Pairwise-parallel gates

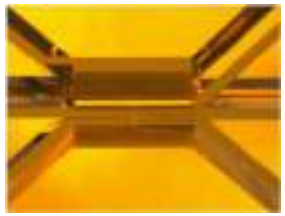
Orthogonal modes



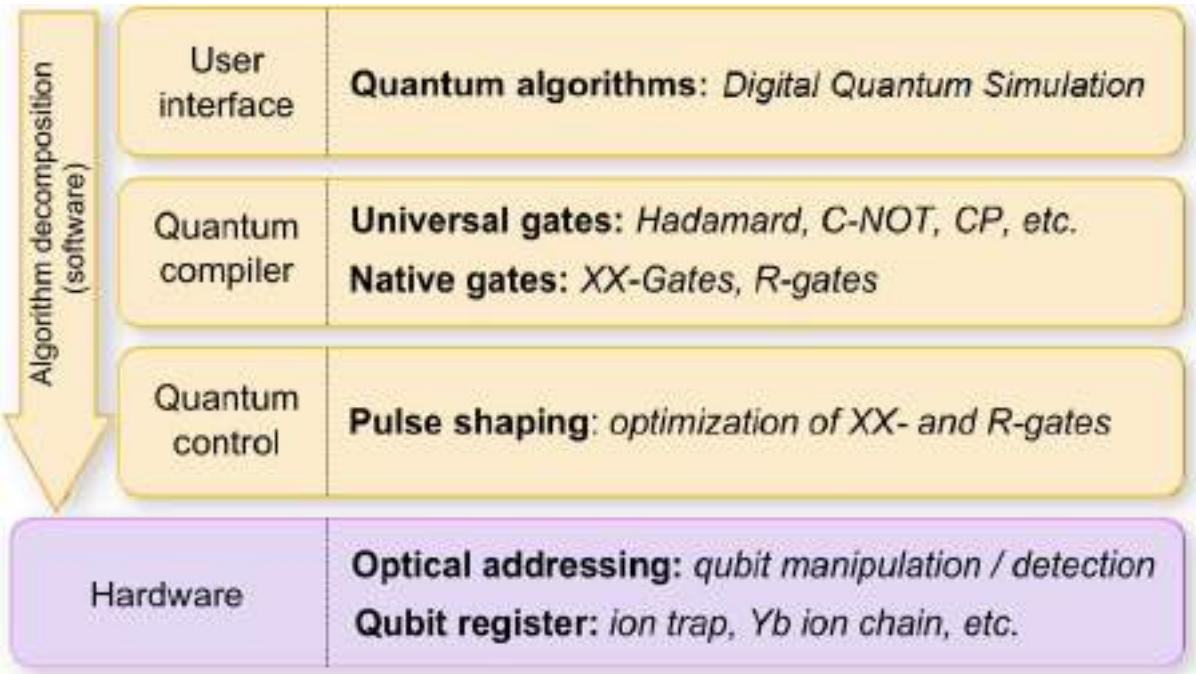
Y. Zhu et al., Adv. Quantum Technol. 020324 (2023).

Outlook

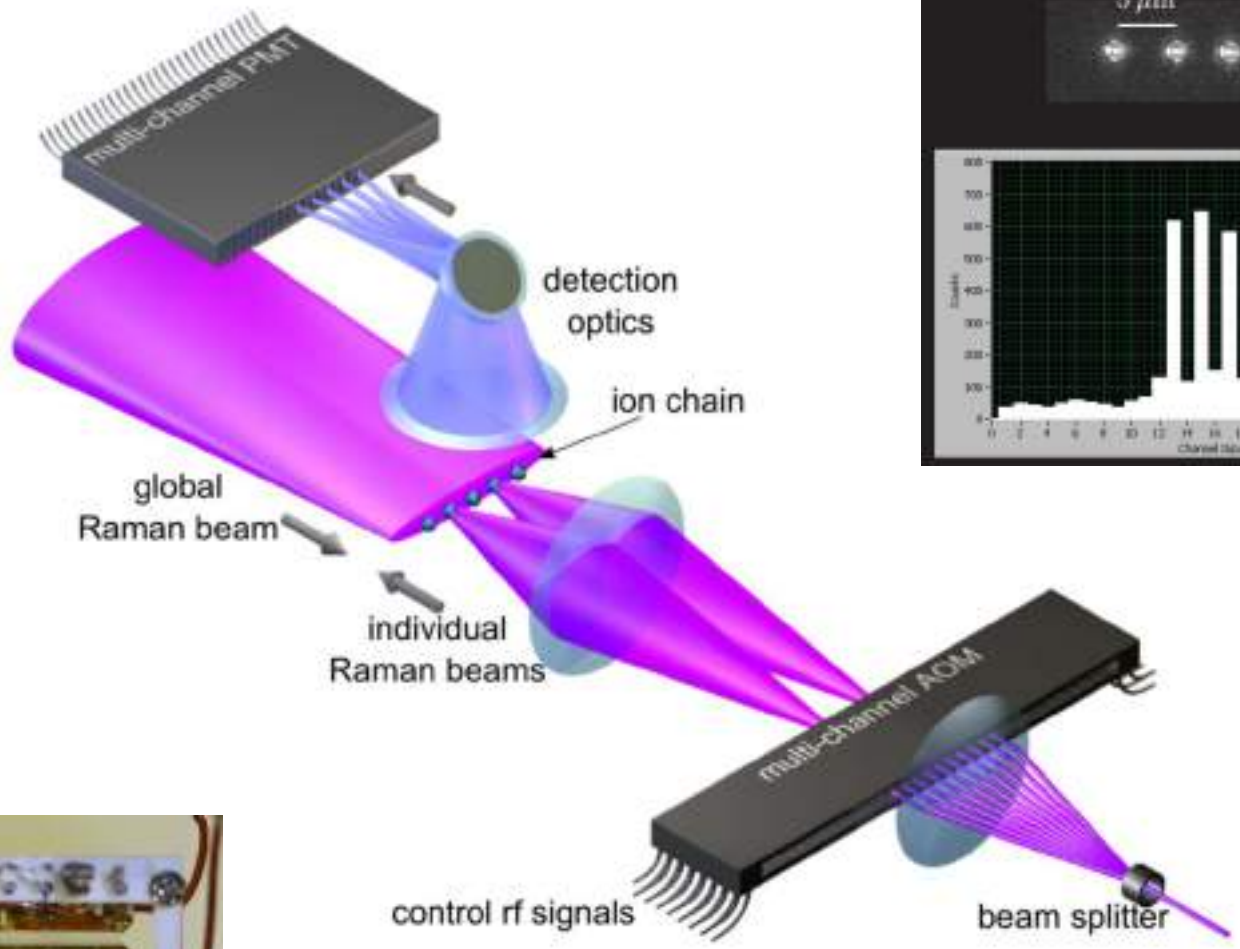
A new monolithic ion trap

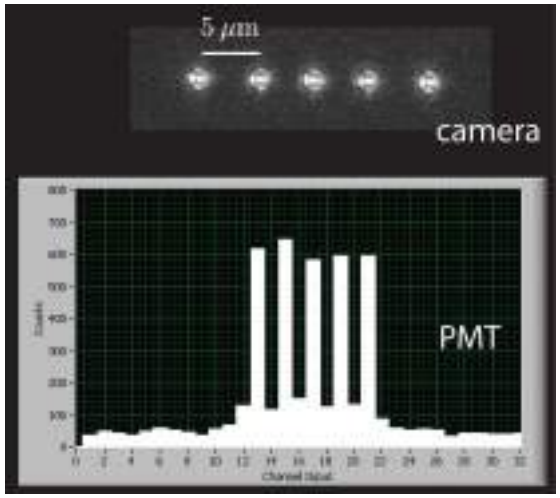


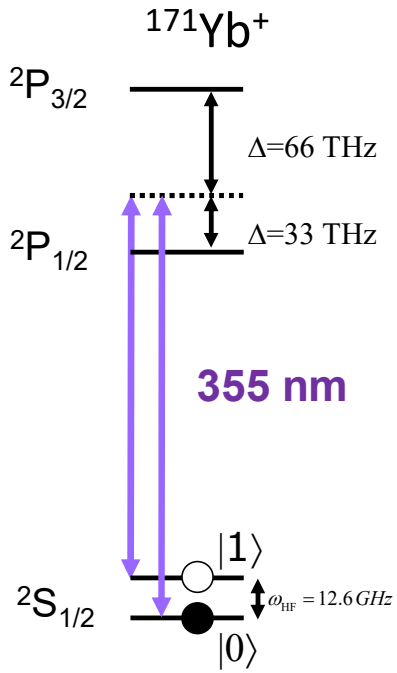
Experimental system: QC architecture

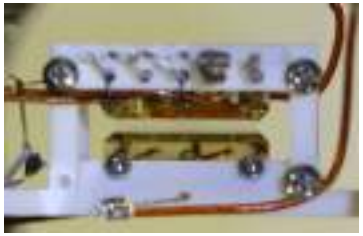


Experimental system: Hardware

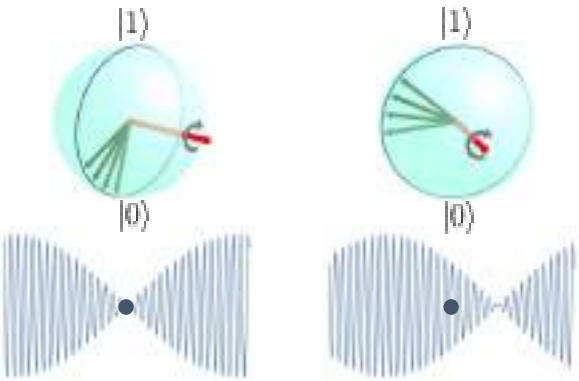




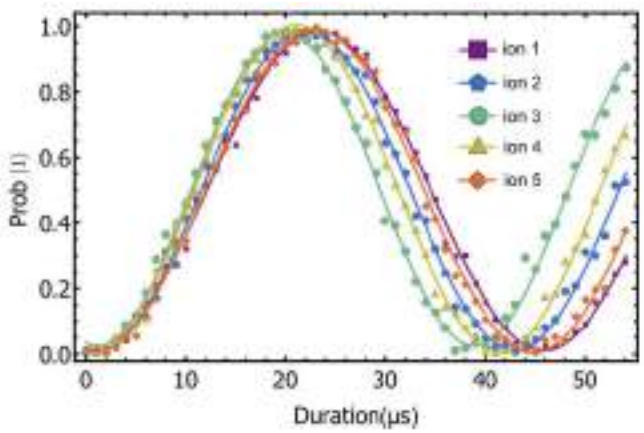


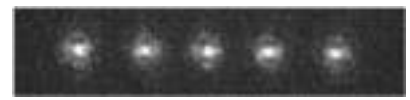


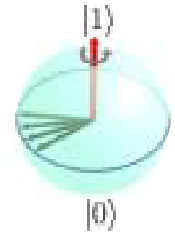
Experimental system: Single qubit gates



Raman beat note







Classical phase shifts

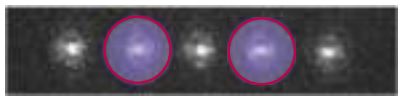
R-gate (*x*/*y* rotations)

$$R_\phi(\theta) = \begin{bmatrix} \cos(\frac{\theta}{2}) & -i\sin(\frac{\theta}{2})e^{-i\phi} \\ -i\sin(\frac{\theta}{2})e^{i\phi} & \cos(\frac{\theta}{2}) \end{bmatrix}$$

***z*-gates**

$$R_z(\theta) = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

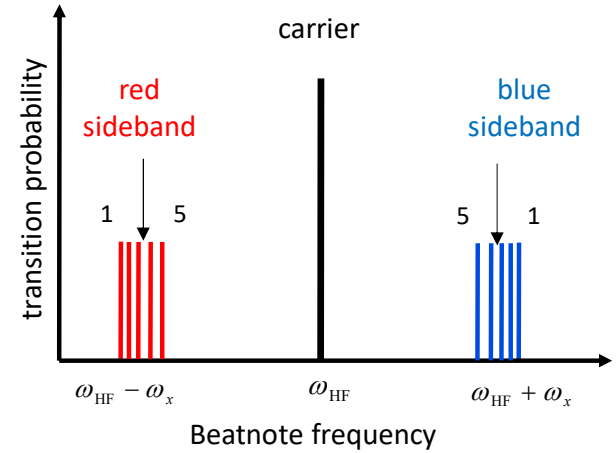
Experimental system: Two-qubit gates



$$U(t) = \exp[-i \sum_{n,k} \hat{D}(\cancel{\alpha_n^k(t)})\sigma_x^n - i \sum_{i,j} \chi_{ij}(t)\sigma_x^i \sigma_x^j]$$

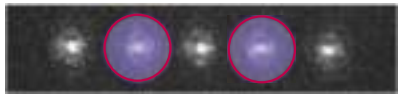
$$XX(\chi_{i,j}) = \begin{bmatrix} \cos(\chi_{i,j}) & 0 & 0 & -i\sin(\chi_{i,j}) \\ 0 & \cos(\chi_{i,j}) & -i\sin(\chi_{i,j}) & 0 \\ 0 & -i\sin(\chi_{i,j}) & \cos(\chi_{i,j}) & 0 \\ -i\sin(\chi_{i,j}) & 0 & 0 & \cos(\chi_{i,j}) \end{bmatrix}$$

$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}(|00\rangle - i|11\rangle)$$



MS gate: PRL **82** (1999)
T. Choi et al. PRL **112**, 19502 (2014)
T. J. Green et al., PRL **114**, 120502 (2015)
P. H. Leung et al. PRL **120**, 020501 (2018)
Y. Shapira et al., PRL **121**, 180502 (2018)

Two-qubit gates



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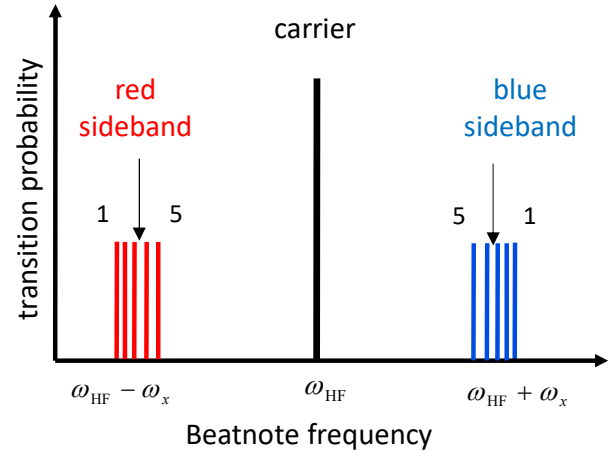
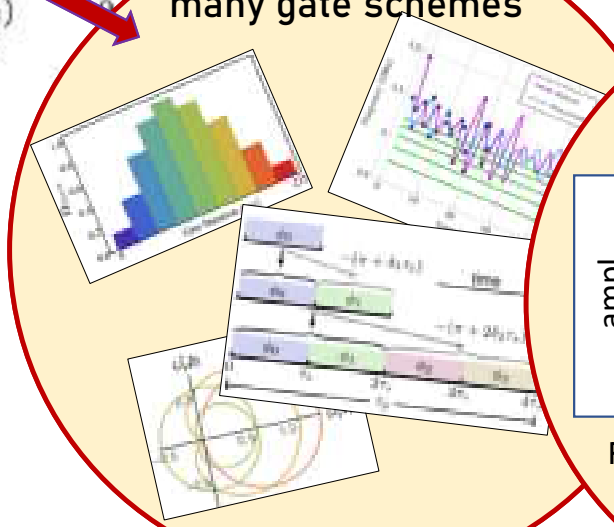


many gate schemes



current favorite:
robust many-tone gate

R. Blümel et. al., PRL 126 (2021)

MS gate: PRL **82** (1999)
T. Choi et al. PRL **112**, 19502 (2014)
T. J. Green et al., PRL **114**, 120502 (2015)
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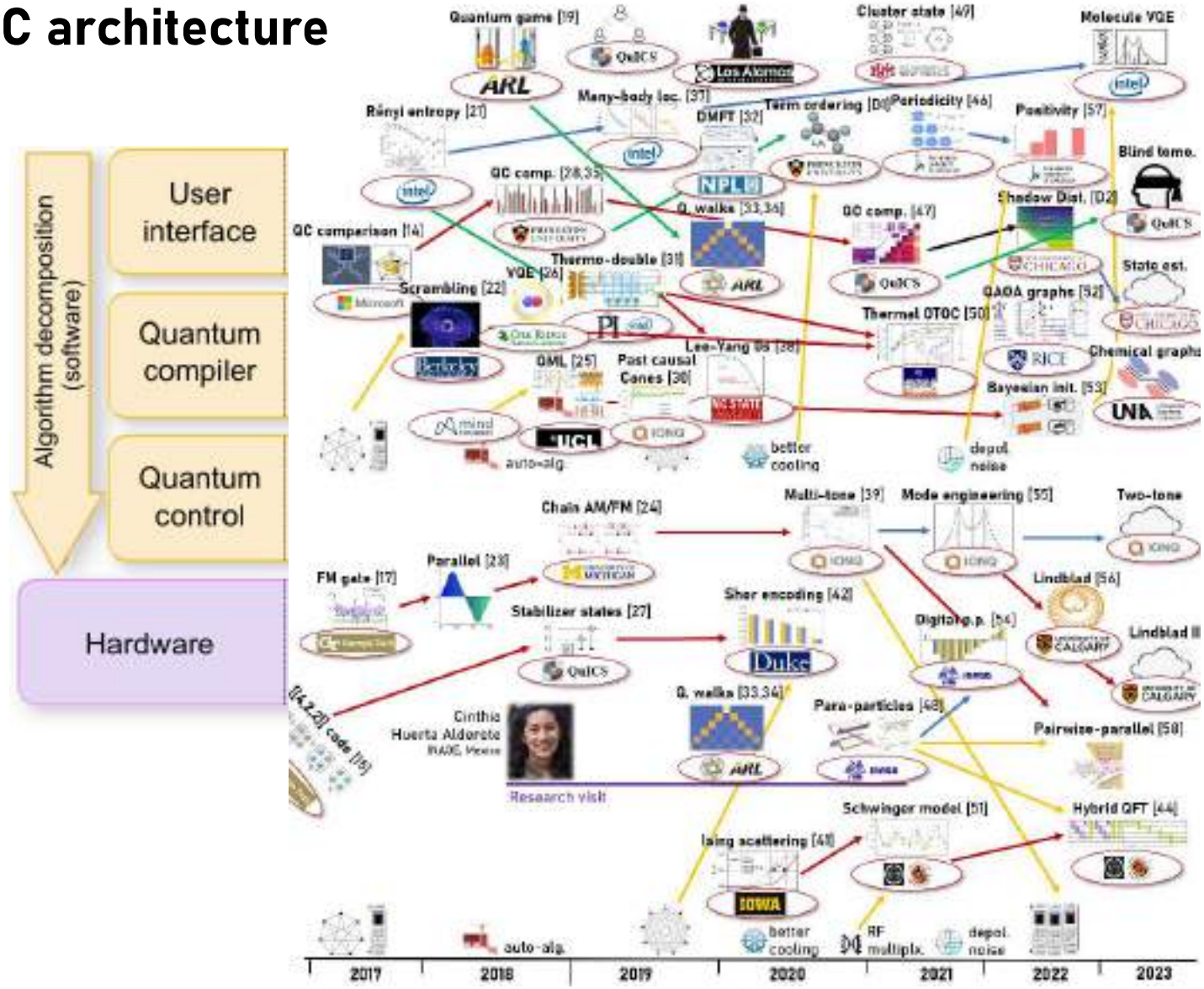
Experimental system: QC architecture



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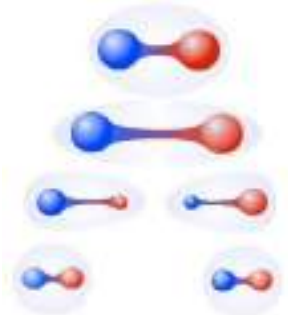
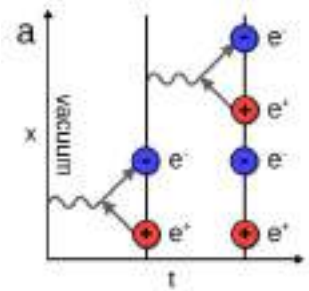
Digital Quantum Simulation: The Schwinger model

Digital Quantum Simulation: The Schwinger model

- Schwinger model is a Quantum Field Theory in 1+1D
- Schwinger model has Quantum Chromodynamics -like phenomena [1,2]
  - Pair creation-annihilation
  - String breaking
- Testbed for quantum simulation methods[3,4,5]

- Digital simulation with long(-ish) time dynamics



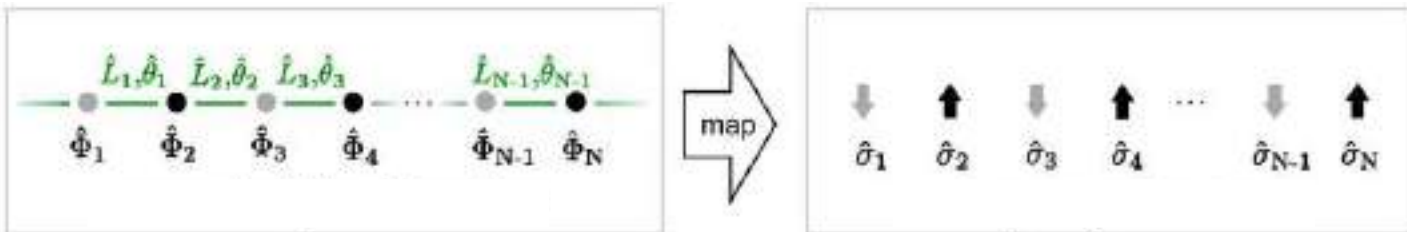


[1] Coleman Ann. Phys. 101 (1976)
[2] Hebenstreit et al PRL. 111 (2013)
[3] Martinez et al Nature 534 (2016)
[4] Surace et al PRX 10 (2020)
[5] Mil et al Science 367 (2020)

Digital Quantum Simulation: The Schwinger model

Lattice Schwinger model (spinless 1+1D QFT, discretized space, normalize)

$$\hat{H}_{\text{lat}} = -\mathrm{i}w \sum_{n=1}^{N-1} [\hat{\Phi}_n^\dagger e^{\mathrm{i}\hat{\theta}_n} \hat{\Phi}_{n+1} - \text{H.C.}] + m \sum_{n=1}^{N} (-1)^n \hat{\Phi}_n^\dagger \hat{\Phi}_n + J \sum_{n=1}^{N-1} \hat{L}_n^2,$$



C. Muschik et al New J. Phys. 19 103020 (2018)

Digital Quantum Simulation: The Schwinger model

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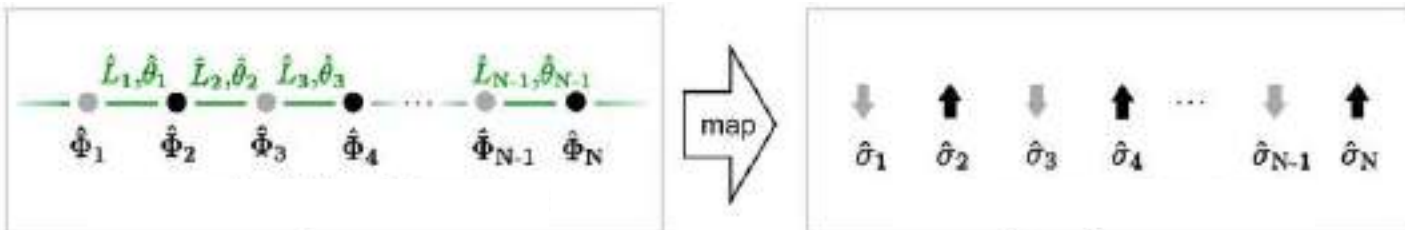


Odd lattice sites:

● ≅ vac ≅ ↑ $\hat{L}_n = \hat{L}_{n-1}$

● ≅ e⁺ ≅ ↓ $\hat{L}_n = \hat{L}_{n-1} - 1$

Even lattice sites:

● ≅ e⁻ ≅ ↑ $\hat{L}_n = \hat{L}_{n-1} + 1$

● ≅ vac ≅ ↓ $\hat{L}_n = \hat{L}_{n-1}$

Gauss' law applies for photon link:

-> number of spin-up qubits conserved

C. Muschik et al New J. Phys. 19 103020 (2018)

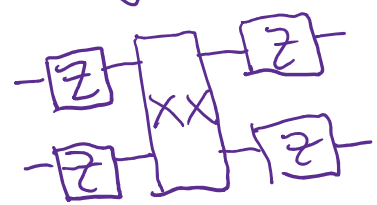
Digital Quantum Simulation: The Schwinger model

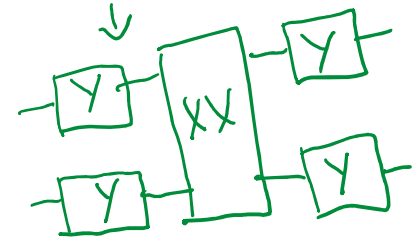
Final qubit Hamiltonian

Fermion mass, hopping on lattice, E-field interaction

$$\hat{H}_s = \frac{\mu}{2}\sum_{n=1}^{N}(-1)^n\sigma_n^z + x\sum_{n=1}^{N-1}\{\sigma_n^+\sigma_{n+1}^- + \text{h.c.}\} + \frac{1}{4}\sum_{n=1}^{N-1}\left\{\sum_{m=1}^{n}\left[\sigma_m^z + (-1)^m\right]\right\}^2$$

$$= \sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y = \sum_n\sum_m \sigma_n^z\sigma_m^z$$





parameters

$x = 0.6, \quad \mu = 0.1, \quad \delta t = 0.5, \quad N_t = 20.$

Run Hamiltonian evolution as a quantum circuit: Trotterization

$$\hat{U} = e^{-i\hat{H}t/\hbar} = \lim_{n\to\infty}(\Pi_k e^{-i\hat{H}_k t/n\hbar})^n$$

First-order Trotter approximation: pick finite n

$$\delta t = t/n \quad \hat{U}_k = e^{-i\hat{H}\delta t/\hbar}$$

(steps don't commute -> term ordering)

Digital Quantum Simulation: The Schwinger model

Final qubit Hamiltonian

Fermion mass, hopping on lattice, E-field interaction

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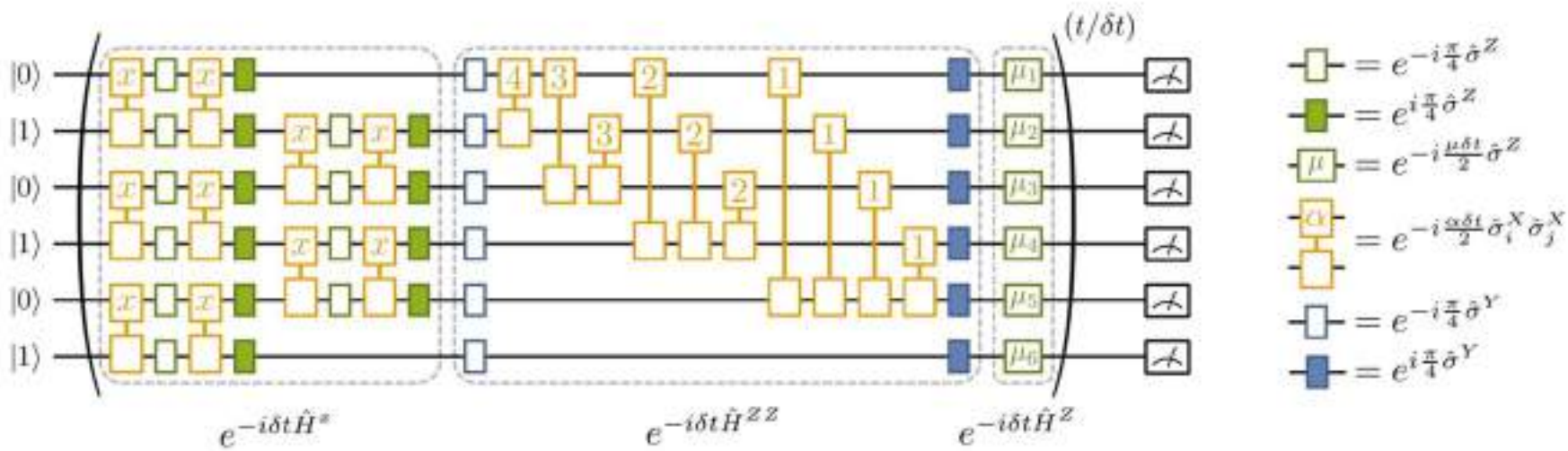
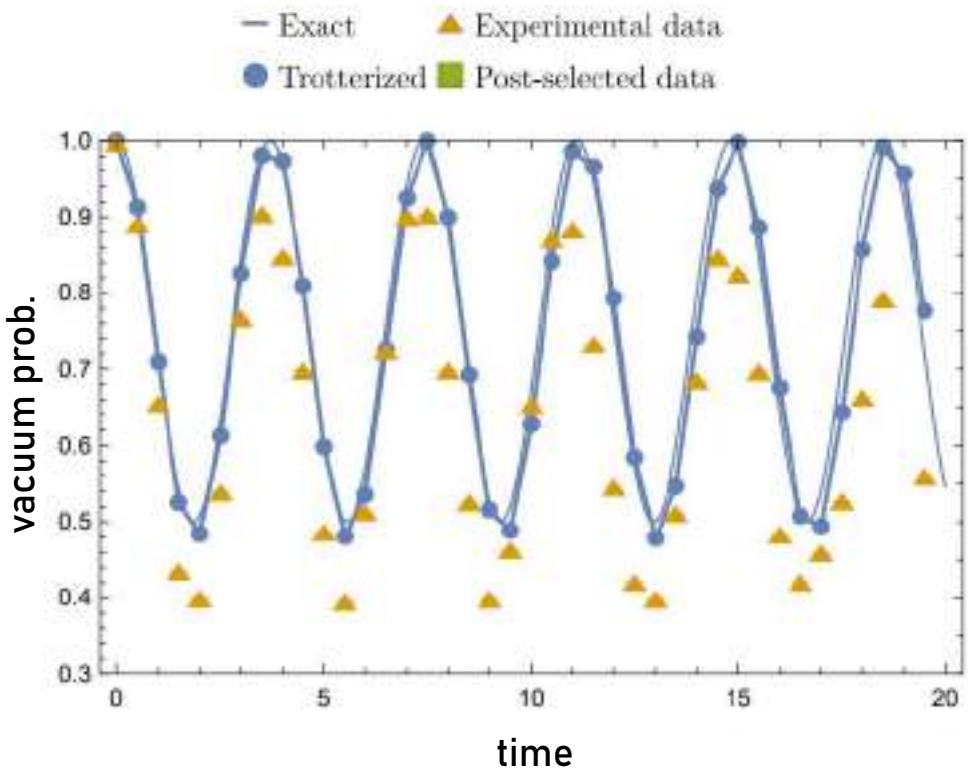
- Start at ground state of x=0
- Evolve with x≠0 to time t (Trotterized) $|\text{vac}\rangle = |0101...01\rangle$
- Measure vacuum survival prob. / particle number density / E-field density



Digital Quantum Simulation: The Schwinger model

Results (1-site model, 2 qubits)

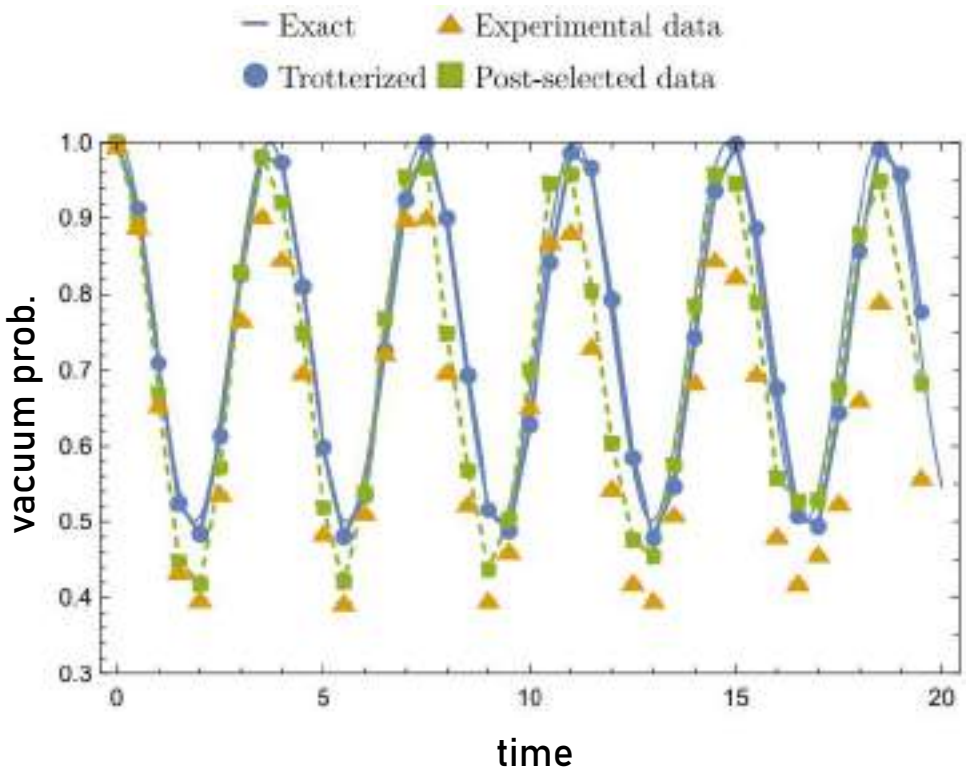


N. H. Nguyen et al., PRX Quantum 3, 020324 (2022)

see also Innsbruck group: E. Martinez, Nature 516, 534 (2016)

Digital Quantum Simulation: The Schwinger model

Results (1-site model, 2 qubits)

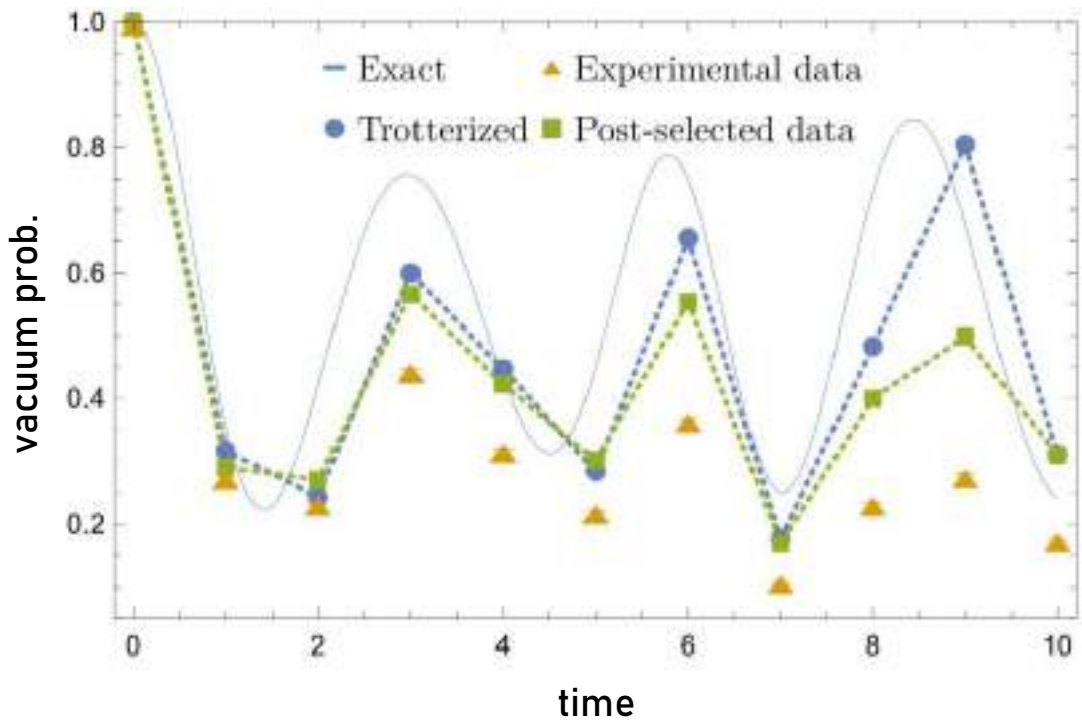


N. H. Nguyen et al., PRX Quantum 3, 020324 (2022)

see also Innsbruck group: E. Martinez, Nature 516, 534 (2016)

Digital Quantum Simulation: The Schwinger model

Results (2-site model, 4 qubits)

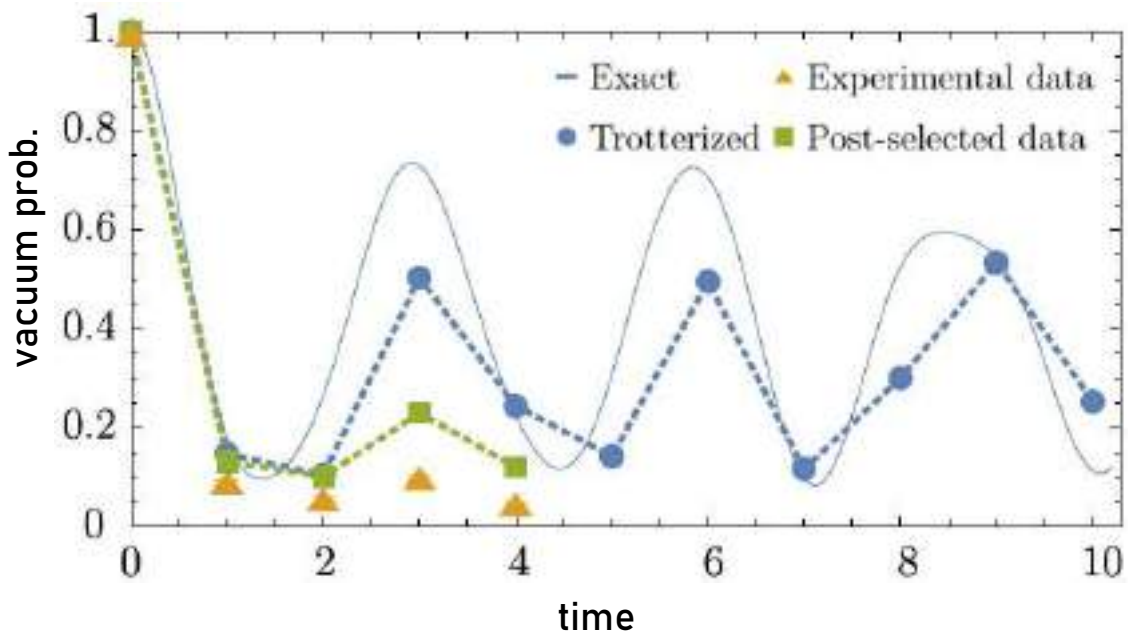


N. H. Nguyen et al., PRX Quantum 3, 020324 (2022)

note term ordering (Trotter error): A. Childs et al., Phys. Rev. Lett. 123, 050503 (2019)

Digital Quantum Simulation: The Schwinger model

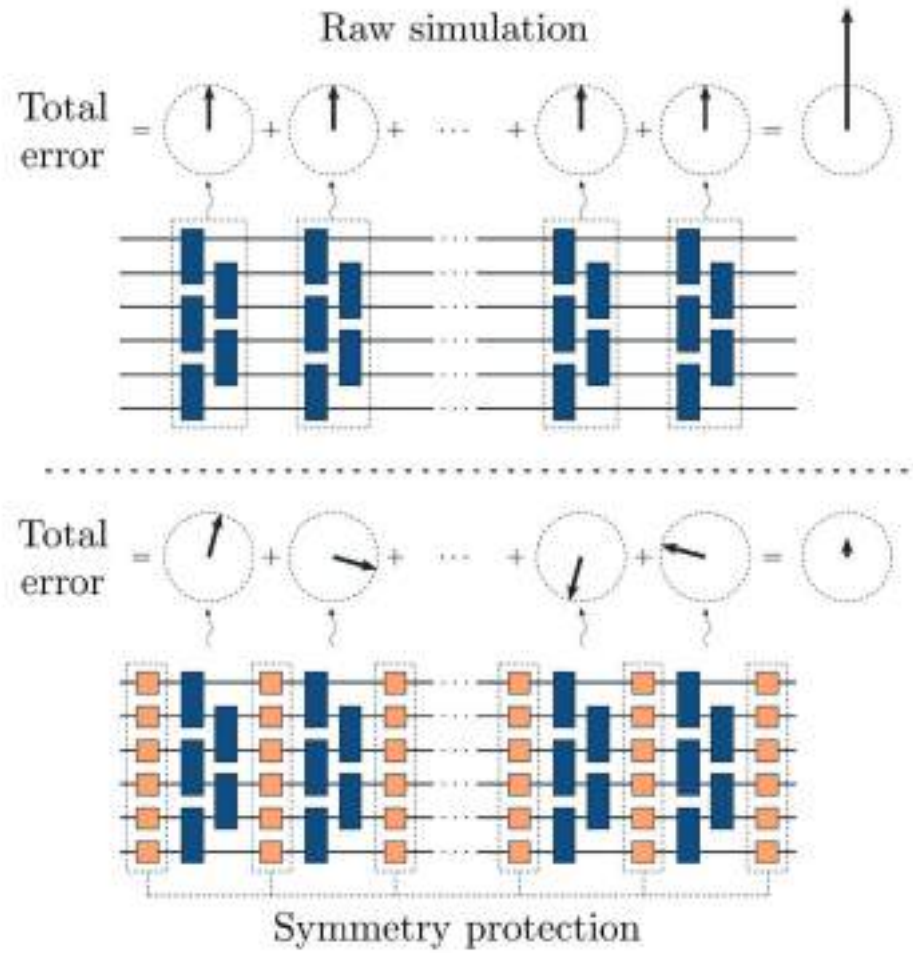
Results (3-site model, 6 qubits)

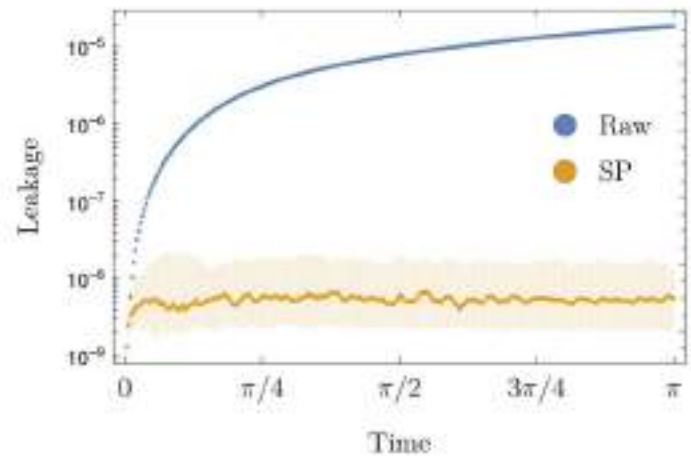


N. H. Nguyen et al., PRX Quantum 3, 020324 (2022)

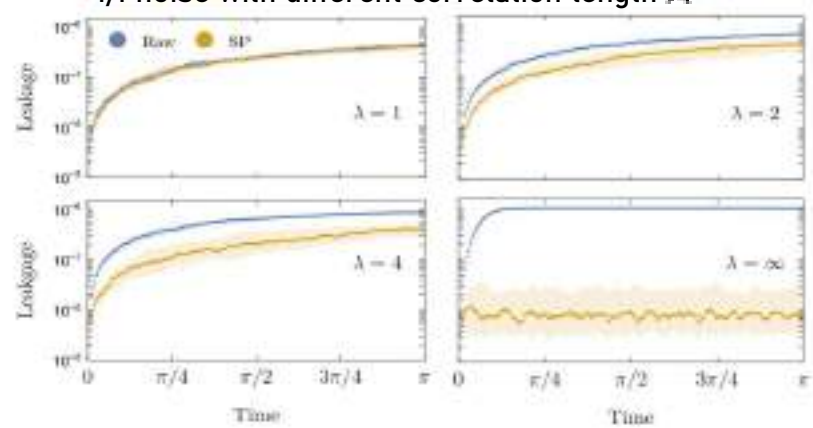
Digital Quantum Simulation: Symmetry Protection

Can we mitigate more of the errors?



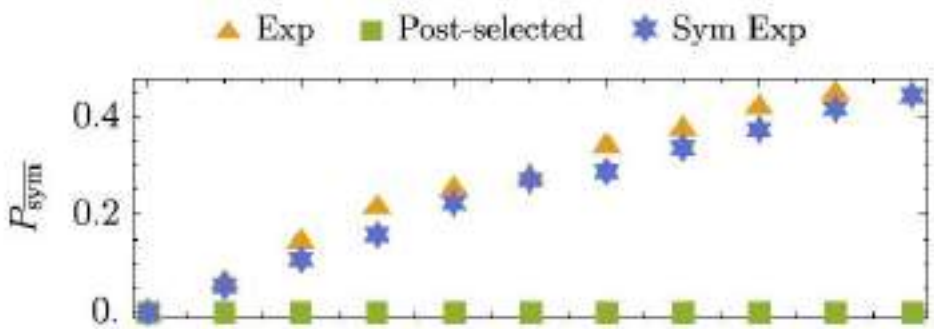


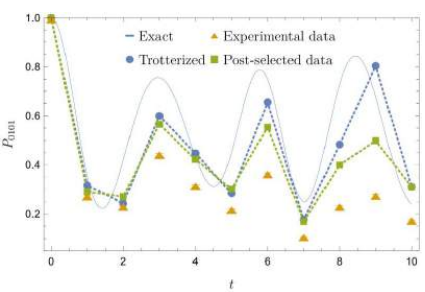
1/f noise with different correlation length λ



M. Tran et al. PRX Quantum 2, 010323 (2021)

Digital Quantum Simulation: Symmetry Protection

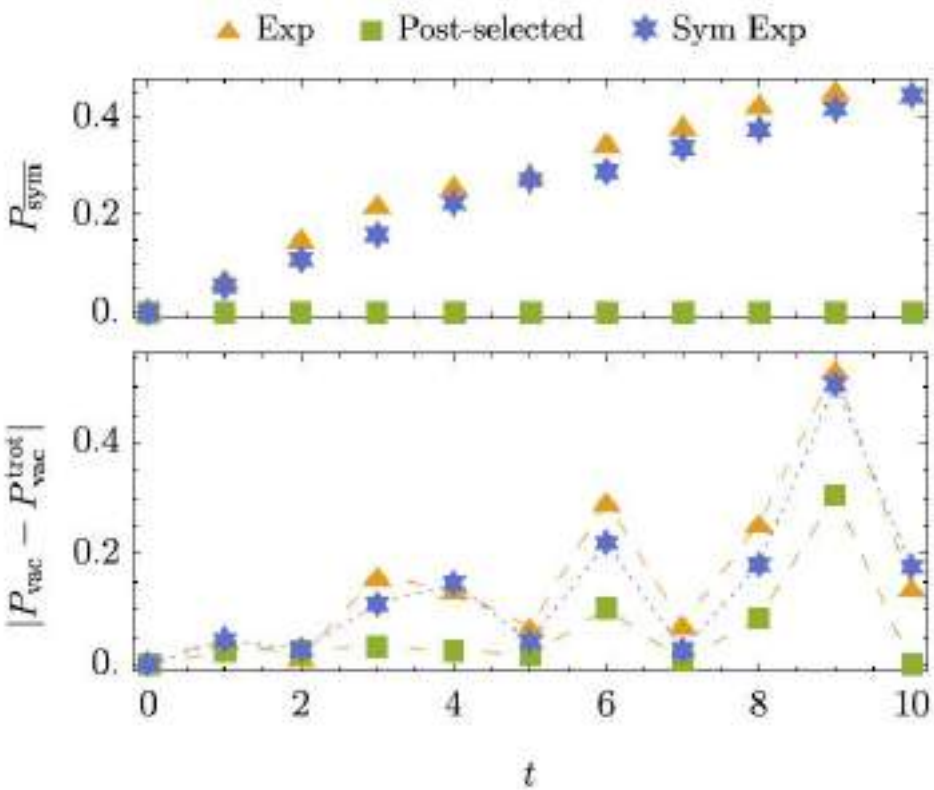


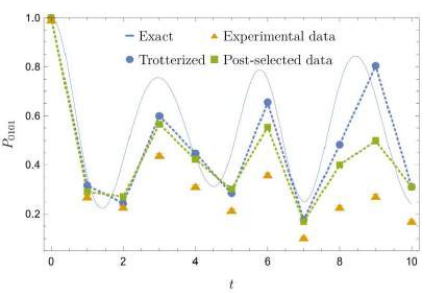


N. H. Nguyen et al., PRX Quantum 3, 020324 (2022)

Digital Quantum Simulation: Symmetry Protection

Conclusions:
- post selection is effective
- our errors is (time-)uncorrelated

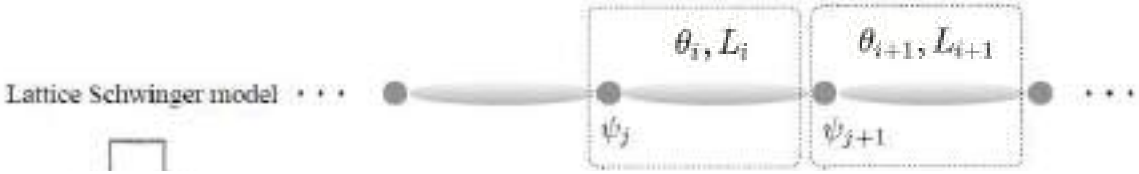




N. H. Nguyen et al., PRX Quantum 3, 020324 (2022)

Difficult Quantum Simulation… such as the Schwinger model

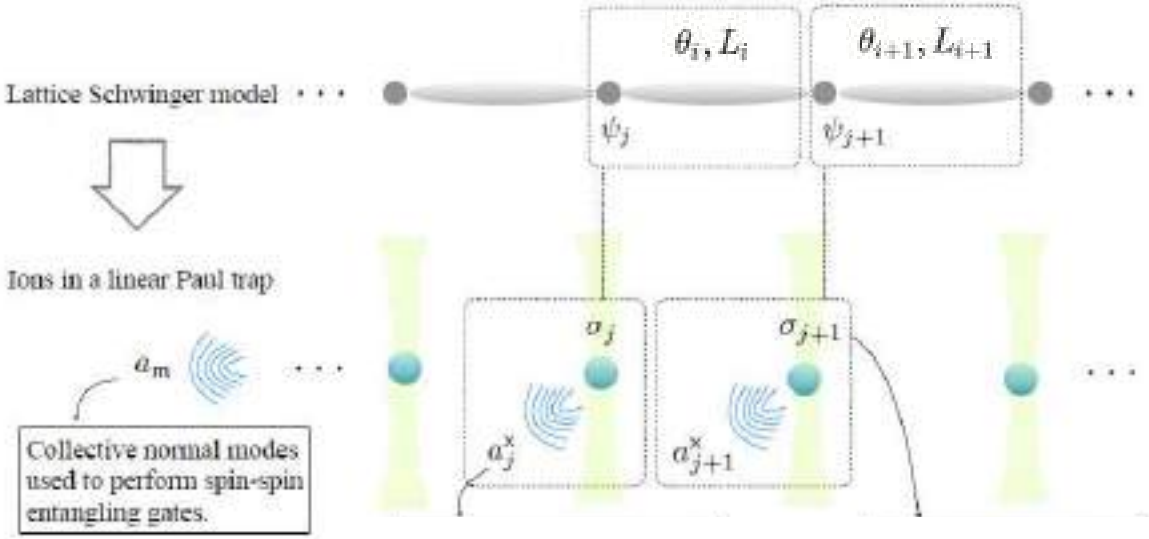
Analog Quantum Simulation Alternative ?



Zohreh Davoudi, NML, G. Pagano, Phys. Rev. Research 3, 043072 (2021)

Hybrid Quantum Simulation

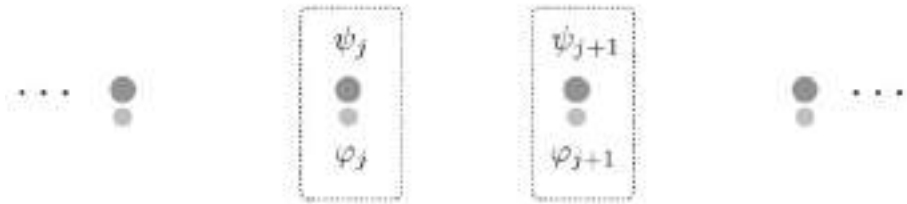
The Schwinger model



Zohreh Davoudi, NML, G. Pagano, Phys. Rev. Research 3, 043072 (2021)

Hybrid Quantum Simulation: The Yukawa model

$$V(r) = -\frac{g^2}{4\pi}\frac{1}{r}e^{-\mu r}$$



- Scalar field theory: scalar bosons (field) interact with fermions (matter)
- Describes the Higgs field interacting with the leptons and quarks
- Explains mass generation

$$H_{\text{Yukawa}} = H^{(I)}_{\text{Yukawa}} + H^{(II)}_{\text{Yukawa}} + H^{(III)}_{\text{Yukawa}}$$

$$H^{(III)}_{\text{Yukawa}} = gb\sum_{j=1}^{N}\psi_j^\dagger\varphi_j\psi_j \qquad \varphi_j = \frac{1}{\sqrt{Nb}}\sum_{k=-N/2}^{N/2-1}\frac{1}{\sqrt{2\varepsilon_k}}(d_k^\dagger e^{-i2\pi kj/N} + d_k e^{i2\pi kj/N})$$

$$H^{(II)}_{\text{Yukawa}} = \sum_{k=-N/2}^{N/2-1}\varepsilon_k\left(d_k^\dagger d_k + \frac{1}{2}\right) \qquad \varepsilon_k = \sqrt{(\tfrac{2\pi k}{Nb})^2 + m_\varphi^2}$$

$$H^{(I)}_{\text{Yukawa}} = \sum_{j=1}^{N}\left[\frac{i}{2b}(\psi_j^\dagger\psi_{j+1} - \psi_{j+1}^\dagger\psi_j) + m_\psi(-1)^j\psi_j^\dagger\psi_j\right]$$

Hybrid Quantum Simulation: The Yukawa model

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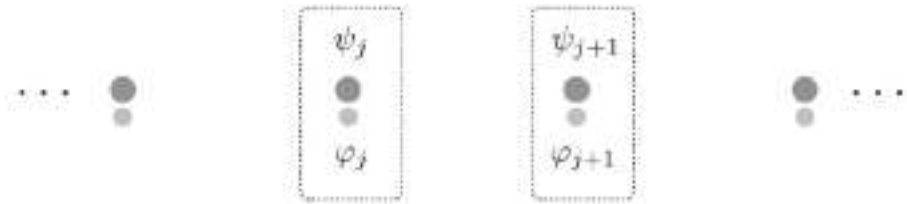

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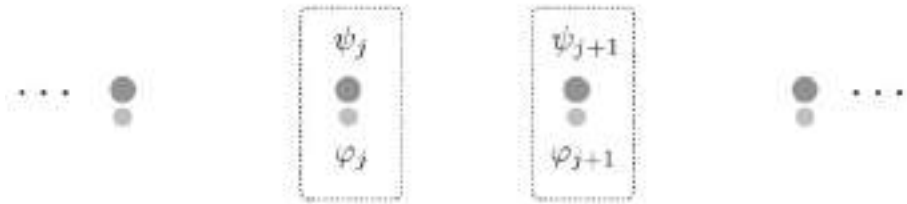

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ancilla ion

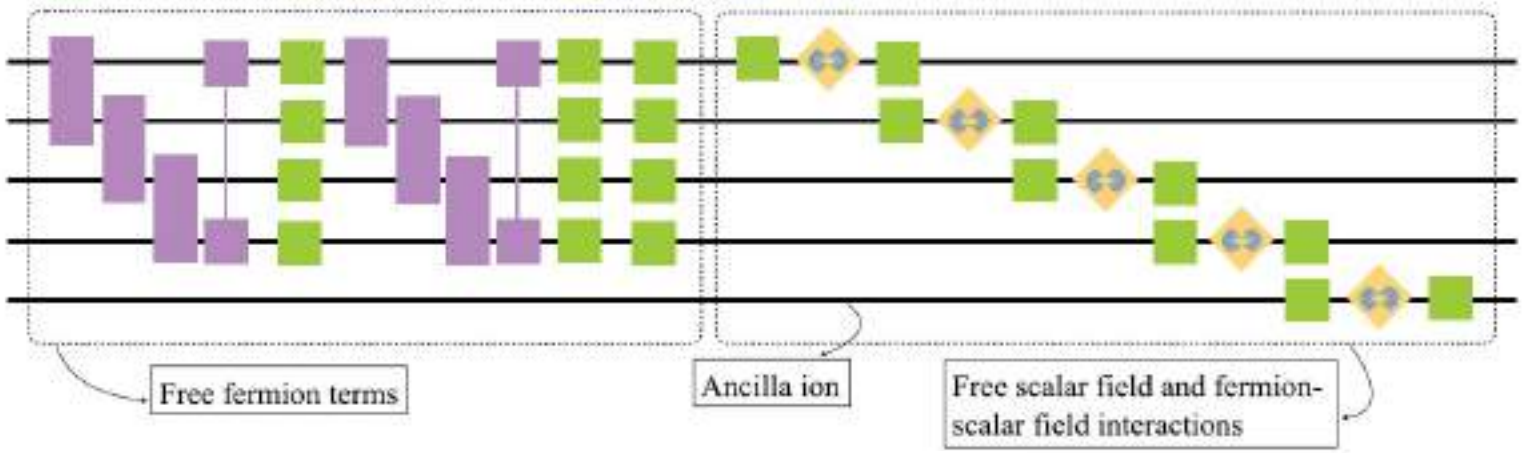
red & blue sideband (rwa, detuned)

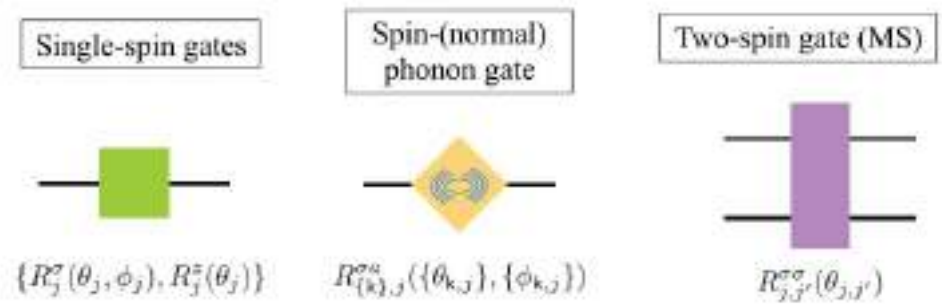
Hybrid Quantum Simulation: The Yukawa model

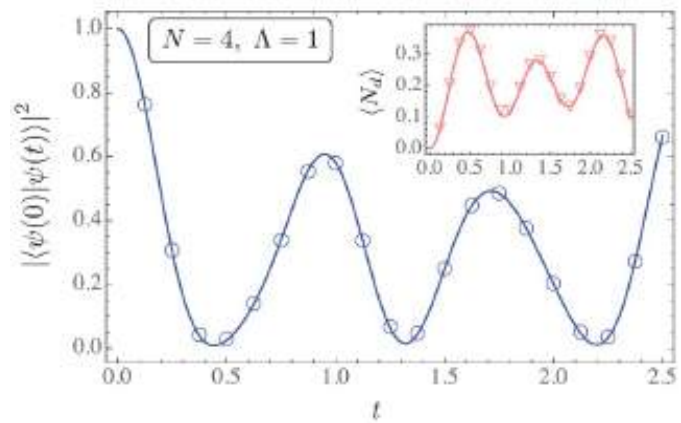
Nhung Nguyen

Monday poster 27

Map fermions to spins and bosons to phonons







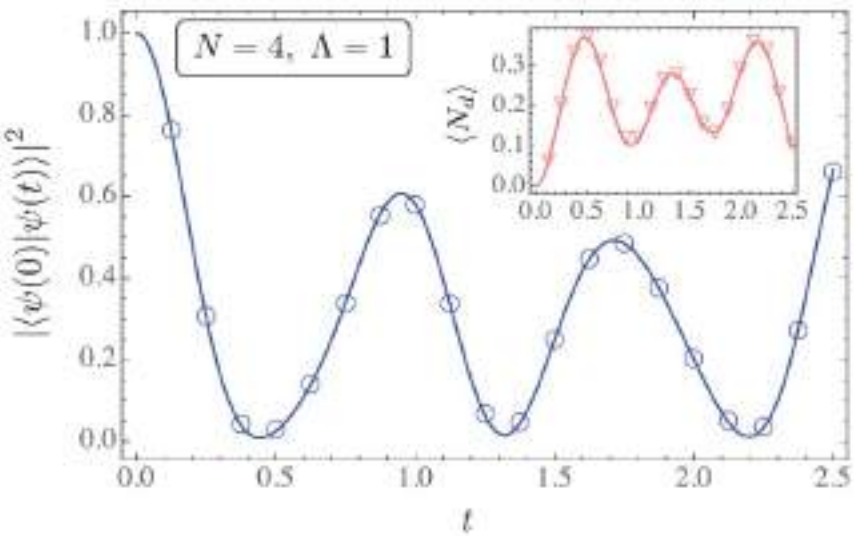
Z. Davoudi, NML, G. Pagano, Phys. Rev. Research 3, 043072 (2021)

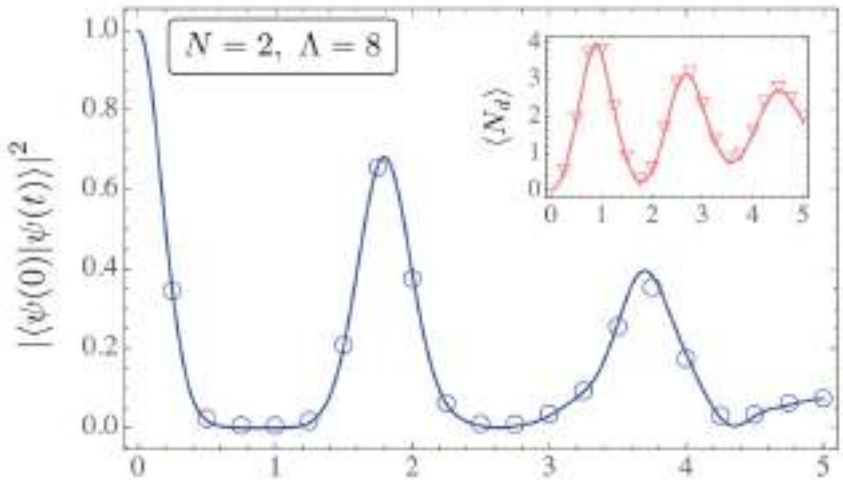
Hybrid Quantum Simulation: The Yukawa model

$$\psi(0)$$

- Start in the ground state of non-interacting H (no fermions, anti-fermions, or bosons)
- Time evolution with Trotterization
- Measure revivals, avg. boson number

$$|\langle\psi(0)|\psi(t)\rangle|^2 \qquad \langle N_d\rangle \equiv \frac{1}{N}\langle\psi(t)|\sum_{k=-N/2}^{N/2-1} d_k^\dagger d_k|\psi(t)\rangle$$

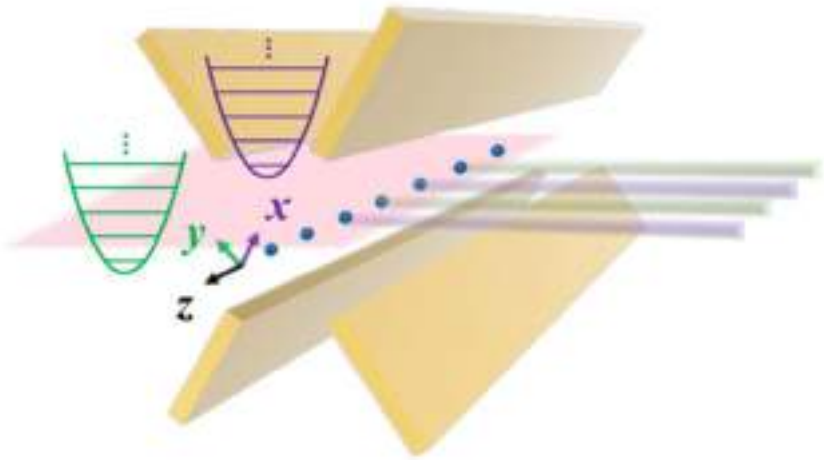




Pairwise parallel gates

Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

Pairwise parallel gates



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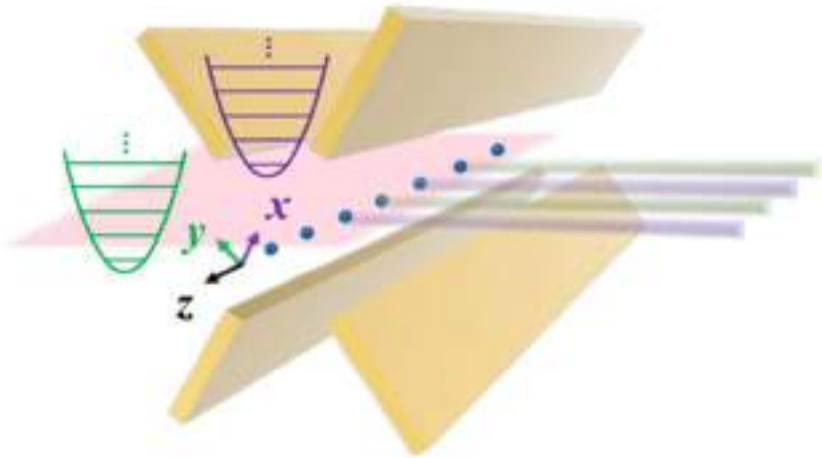
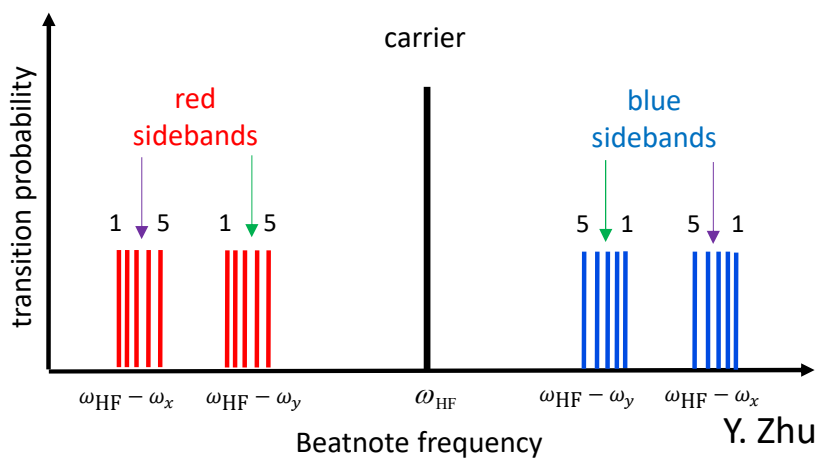
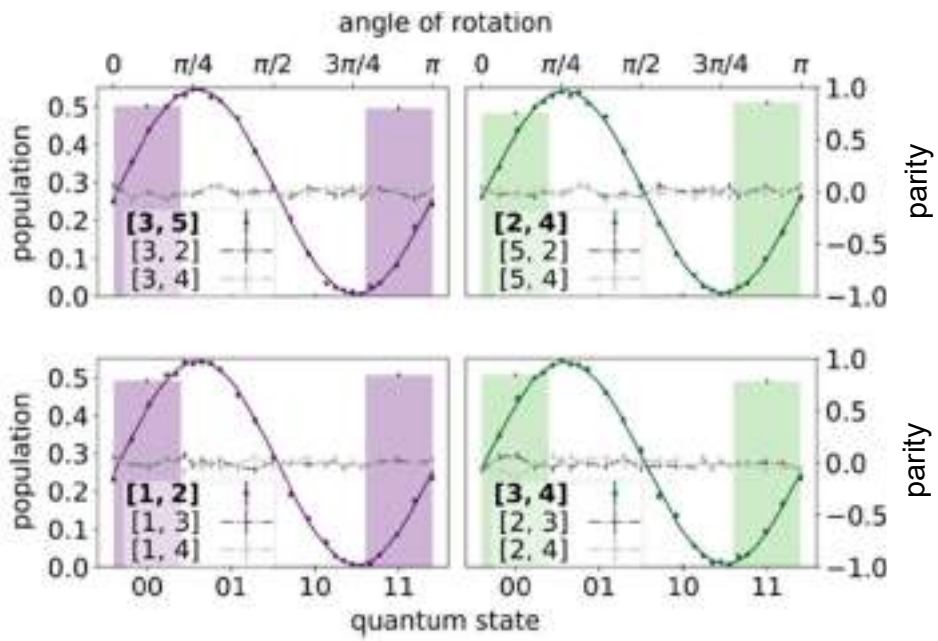




Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

Pairwise parallel gates

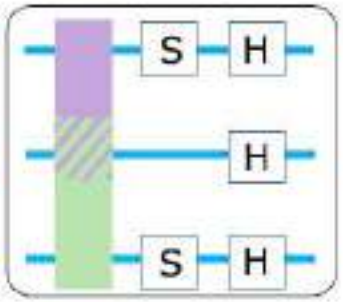
$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}(|00\rangle - i|11\rangle)$$



Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

Pairwise parallel gates

Parallel gates with overlapping ion: GHZ state



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Pairwise parallel gates

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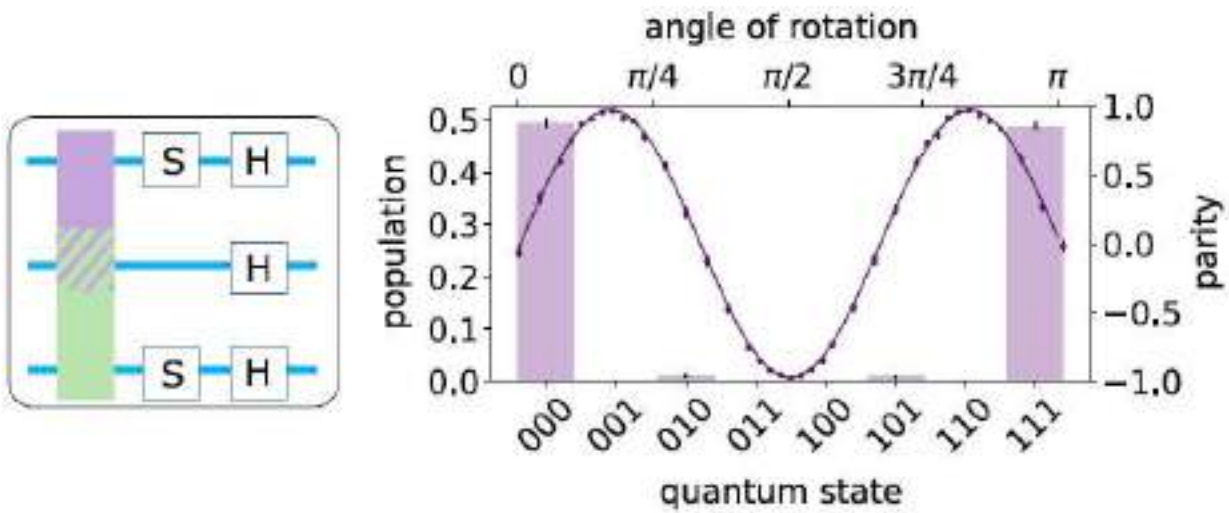
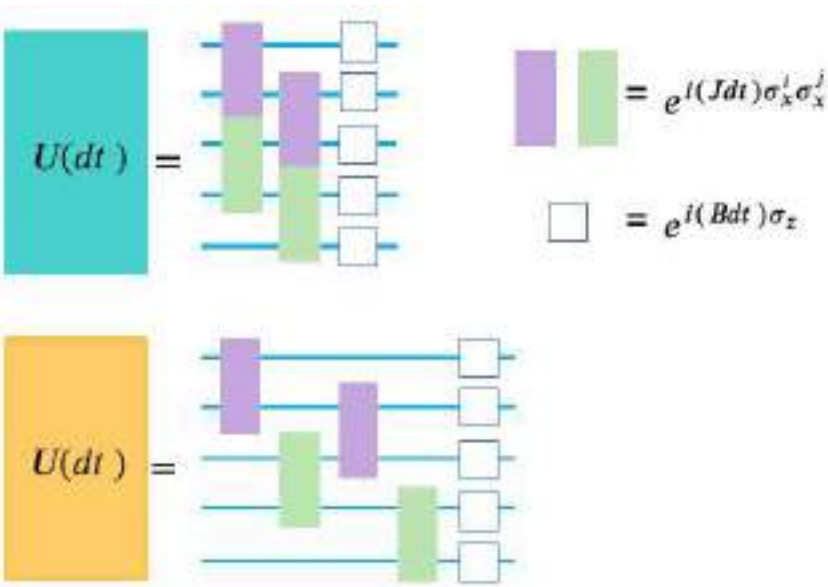


Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

Pairwise parallel gates

Transverse-field Ising model: $$H = -J\sum_{i=1}^{4}\sigma_x^i\sigma_x^{i+1} - B\sum_{i=1}^{5}\sigma_z^i$$



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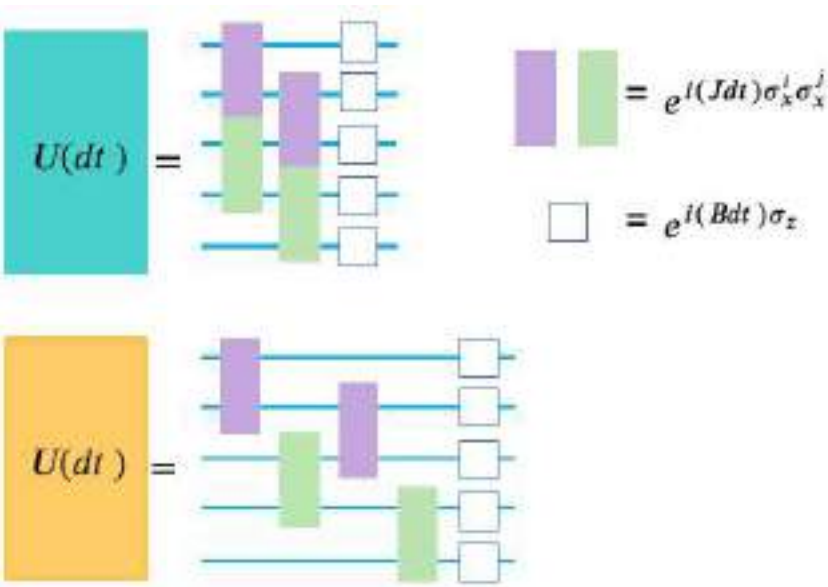
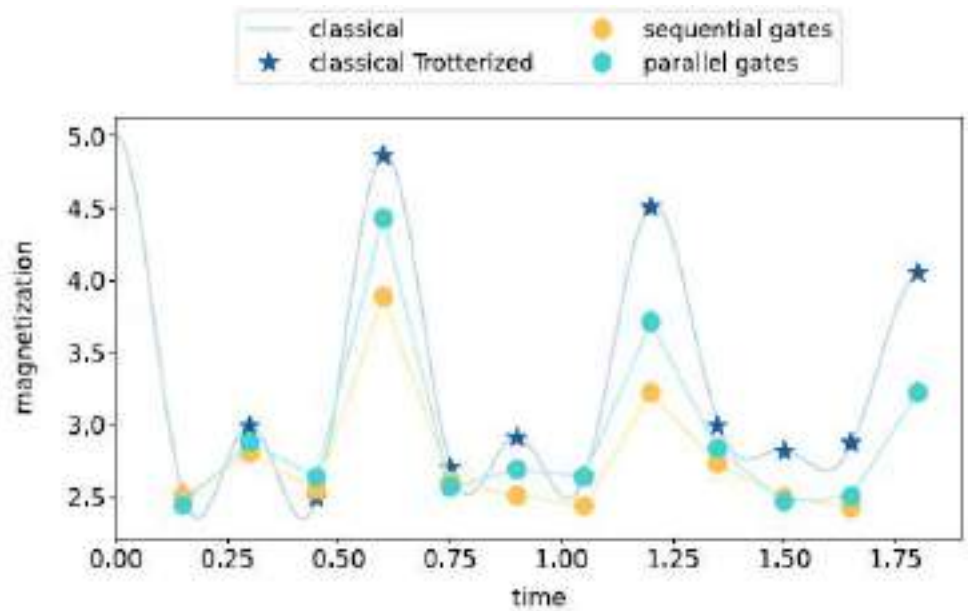




Y. Zhu et al., Adv. Quantum Technol. 020324 (2023)

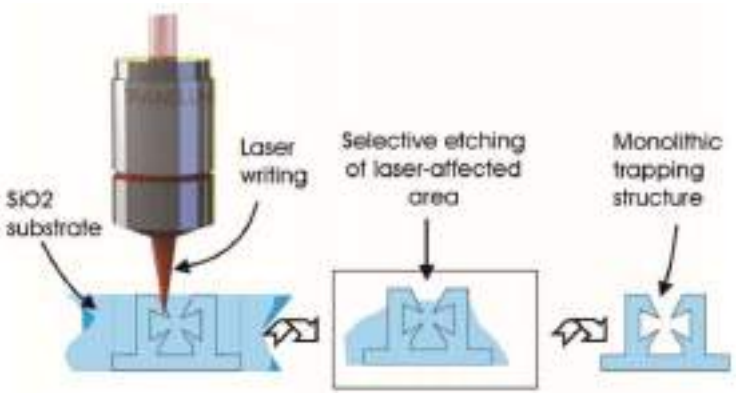
Outlook: A new ion trap platform

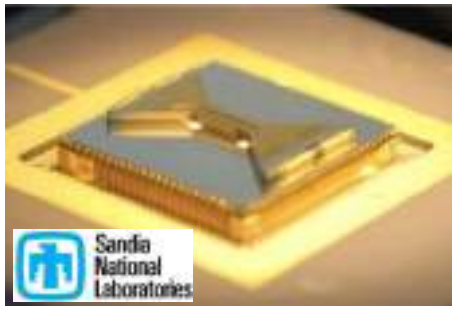
collaboration with G. Pagano (Rice)

Outlook: A new ion trap platform

3D blade trap (with problems)

Monolithic 3D trap made of Fused Silica by Translume Inc.

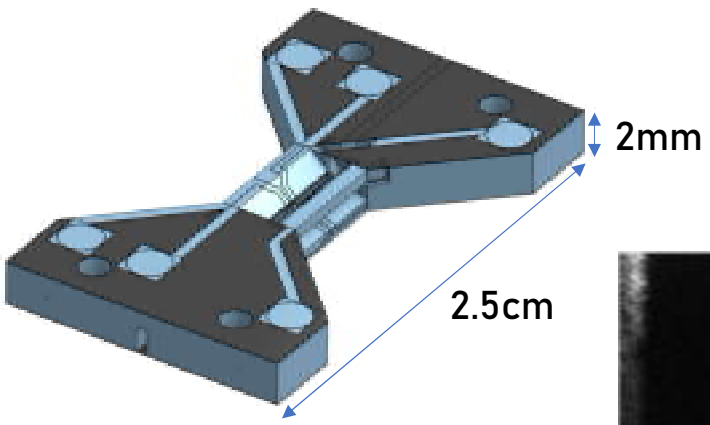


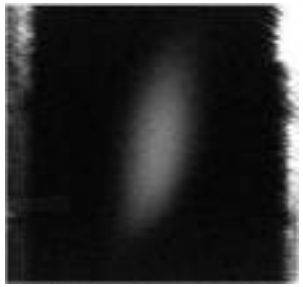


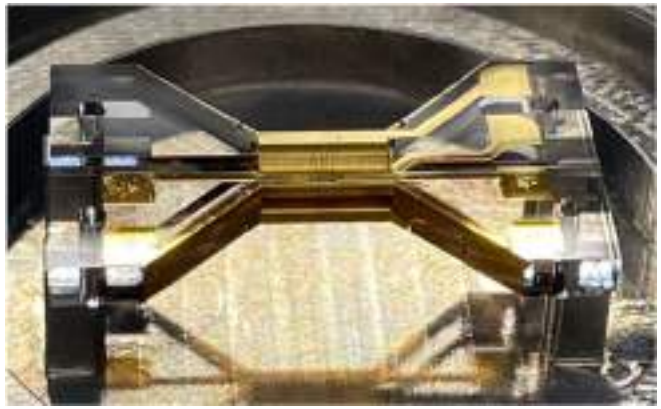
2D surface traps (with fewer/different problems):
DQC groups, PTB, NICT, Honeywell, NIST etc.

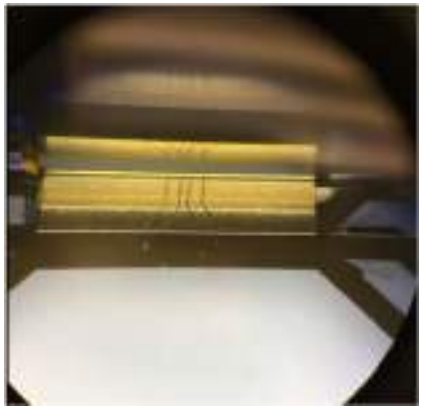
collaboration with G. Pagano (Rice)

Outlook: A new ion trap platform



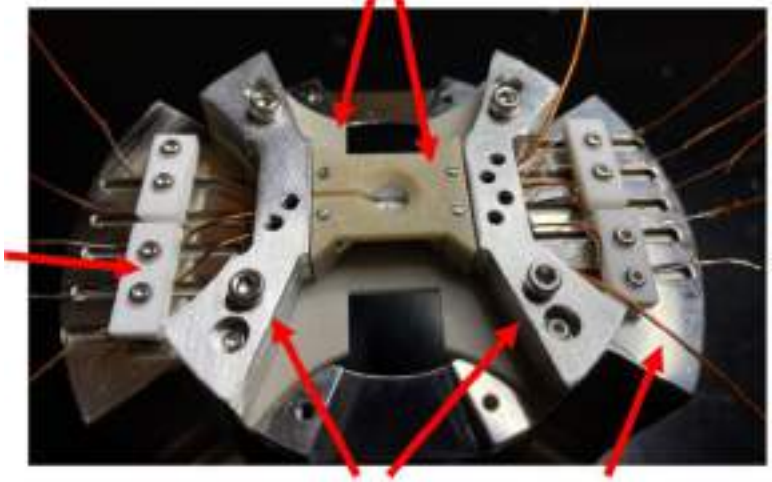






TAMOS to be est. Oct. 2023

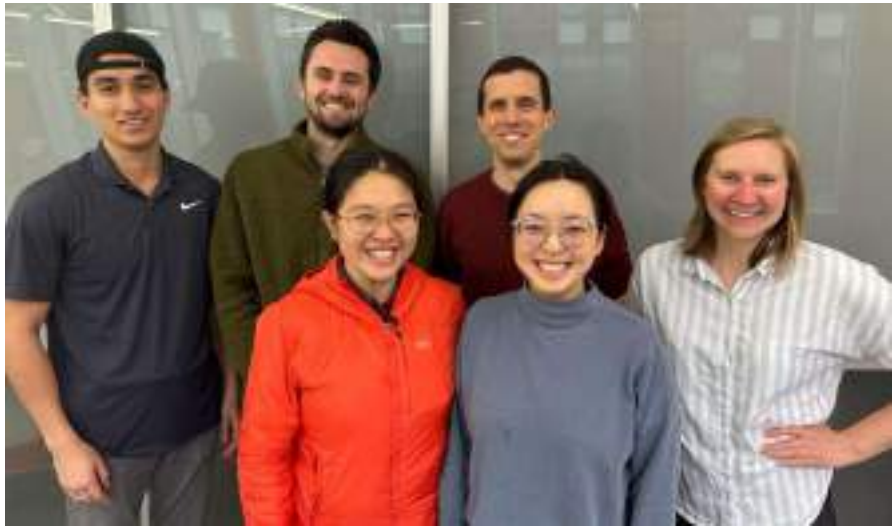
R. Zhuravel G. Pagano NML



jqi UNIVERSITY OF MARYLAND

Tuesday poster 19

Elijah Mossman (u) | Liam Jeanette | NML | Alaina Green



Nhung Nguyen | Yingyue Zhu

Monday poster 27

Duke Quantum Center

Denton Wu | Minseo Kim (u) | Devon Valdez | Henry Luo | Yuanheng Xie | Jiqing Fan (u) | NML

Michael Straus | Mika Chmielewski | Xinyi Dai | Sophie Decoppet | Ana Ferrari

Monday poster 18

QUANTUM SYSTEMS ACCELERATOR

Catalyzing the Quantum Ecosystem

Institute for Robust Quantum Simulation

Duke Quantum Center

Monday posters: 152 (M. Donofrio), 163 (S. Patel), 177 (K. Ranawat), 147 (E. Reed), 176 (J. Whitlow)
Tuesday posters: 60 (J. O'Reilly), 209 (D. Biswas), 113 (J. Toast)


Brown


Calderbank


Cetina


Kim


Klco


Kozhanov


Linke


Marvian


Monroe


Noel